MATHCOUNTS®
COMPETITION SERIES

EST. 1983

2020 - 2021
School Handbook

CHECK OUT
THIS YEAR'S MATH
PROBLEMS ON
PG.12!

2020-2021
MATHCOUNTS®
School Handbook

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WestEd has recognized MATHCOUNTS as having one of the nation's most effective STEM learning programs, listing the Math Video Challenge as an Accomplished Program in STEMworks.

The National Association of Secondary School Principals has placed all three MATHCOUNTS programs on the NASSP Advisory List of National Contests and Activities for 2020-2021.

How to Use This School Handbook

IF YOU'RE A NEW COACH

Welcome! We're so glad you're a coach this year. Please read the **New Program Info for 2020-21** starting on the next page, then check out the **Guide for New Coaches** starting on page 4.

IF YOU'RE A RETURNING COACH

Welcome back! Thank you for coaching again. Please read the **New Program Info for 2020-21** starting on the next page, then access this year's **Handbook Materials** on page 9.

NEW PROGRAM INFO FOR 2020-21

Welcome to the MATHCOUNTS® Competition Series! The safety and wellbeing of our students, coaches, volunteers and members of the MATHCOUNTS community are our top priority, which means the Competition Series needs to be a little different this year...but still an awesome experience! If you have questions about the program details below, please feel free to contact the MATHCOUNTS national office at info@mathcounts.org.

DUE TO COVID-19, THIS YEAR'S MATHCOUNTS COMPETITION SERIES HAS BEEN MODIFIED.

Created in 1983, the **MATHCOUNTS Competition Series** is a national program that provides students in grades 6-8 the opportunity to compete in live math contests against and alongside their peers. Typically, these competitions are conducted as in-person events, but just for this year, we have decided to conduct online competitions at each level leading up to the National Competition.

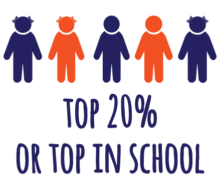
HOW WILL IT WORK? The 2020-2021 Competition Series will have 4 levels of *official* competition—chapter, chapter invitational, state and national—and 4 *unofficial* online practice competitions.

Schools register in the fall and work with students during the year. Students will have the opportunity to take 4 online practice competitions beginning in October. Any number of students from a school can participate in team meetings. Practice competitions will be released October 15, November 15, December 15 and January 22. MATHCOUNTS strongly recommends schools participate in these practice competitions. It is important your students know how to use the competition platform before competition day.

> ***Any student whose school is not participating in the program can register as a non-school competitor (NSC).*** *MATHCOUNTS encourages students to pursue all avenues to participate through their school before registering as NSCs.*

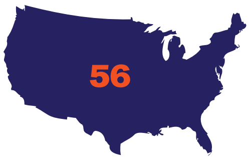
Between 1 and 15 individuals from each school participate in the 2021 Chapter Competition, which will be available online February 5 at 1:00pm ET through February 6 at 1:00pm ET. Coaches determine which students will participate in the Chapter Competition. The competition will be conducted through the Art of Problem Solving (AoPS) Contest Platform, and each student will compete solely as an individual; there will be no team competition.

> *Like school competitors,* ***NSCs*** *will take the Chapter Competition online through the AoPS Contest Platform February 5-6 and will compete as individuals.*



The top scoring student from each school and the top 20% of individuals from each chapter advance to the 2021 Chapter Invitational, taking place online February 25 at 7:00pm ET. Every competitor advancing to this level will be required to take the competition on the AoPS Contest Platform at the same time. Each student will compete solely as an individual; there will be no team competition.

> ***NSCs*** *can only advance to the Chapter Invitational by scoring in the top 20% of their chapter. Like school competitors, NSCs must take the 2021 Chapter Invitational on February 25 at 7:00pm ET on the AoPS Contest Platform.*



Top students from the Chapter Invitational advance to the 2021 State Competition, taking place online March 25 at 7:00pm ET. (State level advancement details will be announced prior to the Chapter Invitational.) Every competitor advancing to this level will be required to take the competition on the AoPS Contest Platform at the same time. Each student will compete solely as an individual; there will be no team competition.

*Advancement to the State Competition will be the same for both school competitors and **NSCs**. NSCs must take the 2021 State Competition on March 25 at 7:00pm ET on the AoPS Contest Platform.*

The top individuals from each state receive an all-expenses-paid trip to the 2021 Raytheon Technologies MATHCOUNTS National Competition, which takes place May 8-11 in Washington, DC. Up to 224 students will be invited to compete for individual and team awards. As of August 2020, this is planned as an in-person event, but subject to change, if necessary.

*Advancement to the national competition will be the same for both school competitors and **NSCs**. An NSC's state will be determined by school location.*

HOW WILL REGISTRATION WORK? Registration this year is a 2-step process:

1. Registering and paying through the MATHCOUNTS Foundation
2. Setting up online competition access through Art of Problem Solving (AoPS)

1 Coaches or school officials should register their schools online at **www.mathcounts.org/compreg**:

- **June 15 – October 1**: Early Bird Registration ($30 per student)
- **October 2 – December 1**: Regular Registration ($35 per student)
- **December 2 – January 15**: Late Registration ($40 per student)

Before registering, non-school competitors (NSCs) must confirm with school officials that their school will not be participating in the Competition Series. MATHCOUNTS (1) may contact the schools of registered NSCs to confirm that the school is not participating in the Competition Series and (2) reserves the right to cancel an NSC's registration, without refund, if their school registers for the Competition Series. NSCs must register online and pay with a credit card at **www.mathcounts.org/nscreg**:

- **August 3 – October 1**: Early Bird Registration ($60 per student)
- **October 2 – December 1**: Regular Registration ($65 per student)
- **December 2 – January 15**: Late Registration ($70 per student)

Schools and NSCs must register and pay by January 15, 2021 to participate. MATHCOUNTS cannot guarantee participation for registrants with outstanding invoices after January 15.

2 Because arrangements must be made for online competitions...

- **All coaches, school competitors and NSCs must create a free account at AoPS.** Coaches at schools will be given instructions to ensure their students are signed up with AoPS and linked to their school.
- **Schools must indicate which of their students will participate in the Chapter Competition by January 15, 2021.**

COMPETITION COACHES:
PLEASE BE SURE TO COMPLETE BOTH PARTS OF THE REGISTRATION PROCESS THIS YEAR!

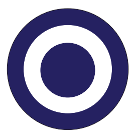
Guide For New Coaches

Welcome to the MATHCOUNTS® Competition Series! Thank you so much for serving as a coach this year. Your work truly does make a difference in the lives of the students you mentor. We've created this Guide for New Coaches to help you get acquainted with the Competition Series and understand your role as a coach in this program.

If you have questions at any point during the program year, please feel free to contact the MATHCOUNTS national office at info@mathcounts.org.

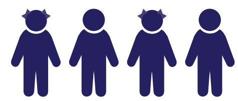
WHAT DOES THE TEST LOOK LIKE? MATHCOUNTS competition typically consists of 4 rounds—Sprint, Target, Team and Countdown Rounds. Altogether the rounds take about 3 hours to complete. **However, Team and Countdown Rounds will not be conducted officially in the 2020-2021 Competition Series until the national level.** Here's what each round looks like.

Sprint Round
40 minutes
30 problems total
no calculators used
focus on speed and accuracy



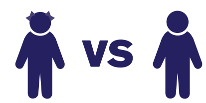
Target Round
Approx. 30 minutes
8 problems total
calculators used
focus on problem-solving and mathematical reasoning

The problems are given to students in 4 pairs. Students have 6 minutes to complete each pair.



Team Round
20 minutes
10 problems total
calculators used
focus on problem-solving and collaboration

National level only for 2020-2021. The online practice competitions will have team rounds.



Countdown Round
Maximum of 45 seconds per problem
no calculators used
focus on speed and accuracy

National level only for 2020-2021.

HOW DO I GET MY STUDENTS READY FOR THESE COMPETITIONS? What specifically you do to prepare your students will depend on your schedule as well as your students' schedules and needs. But in general, working through lots of different MATHCOUNTS problems and completing practice competitions are the best ways to prepare. Each year, MATHCOUNTS provides the *MATHCOUNTS School Handbook* to all coaches, plus lots of additional free resources online. This guide will explain the layout of the *School Handbook* and other resources, plus give you tips on structuring your team meetings and preparation schedule.

THE ROLE OF THE COMPETITION COACH

Your role as the coach is such an important one, but that doesn't mean you need to know everything, be a math expert or treat coaching like a full-time job. Every MATHCOUNTS coach has a different coaching style and you'll find the style that works best for you and your students. But in general, **every good MATHCOUNTS coach must do the following**:

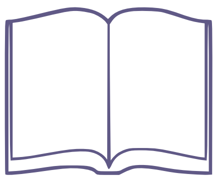
- Schedule and run an adequate number of practices for participating students (these can be online).
- Help motivate and encourage students throughout the program year.
- Select the 1-15 student(s) who will represent the school at the Chapter Competition in February.
- Ensure students can log in on the AoPS Contest Platform and are familiar with using the platform.

You don't need to know how to solve every MATHCOUNTS problem to be an effective coach. In fact, many coaches have told us that they themselves improved in mathematics through coaching. Chances are, you'll learn with and alongside your students throughout the program year.

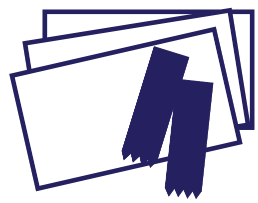
You don't need to spend your own money to be an effective coach. You can prepare your students using solely the free resources and this handbook. We give coaches numerous detailed resources and recognition materials so you can guide your Mathletes® to success even if you're new to teaching, coaching or competition math, and even if you use only the free resources MATHCOUNTS provides all competition coaches.

MAKING THE MOST OF YOUR RESOURCES

As the coach of a registered competition school, you already have received what we at MATHCOUNTS call the **School Competition Kit**. Your kit includes the following materials for coaches.



2020-2021 MATHCOUNTS School Handbook
The most important resource included in the School Competition Kit. Includes 200 problems.



Student Ribbons and Certificates
Ribbons and participation certificates for each registered student.

You'll also get access to electronic resources including **50 more handbook problems**. The following are available to coaches at **www.mathcounts.org/coaches**. This section of the MATHCOUNTS website is restricted to coaches and you already should have received an email with login instructions. *If you have not received this email, please contact us at info@mathcounts.org to confirm we have your correct email address.*

2020 MATHCOUNTS School, Chapter + State Competitions
Released by mid-April 2020
Each level includes all 4 test rounds and the answer key

MATHCOUNTS Practice Plans for Team Meetings
Pre-planned virtual mini-lessons, each 45-60 minutes, that cover a particular math topic.

MATHCOUNTS Problem of the Week
Released each Monday
Each multi-step problem relates to a timely event

You can use any or all of these resources to choose the students who will represent your school at the Chapter Competition.

It is especially important though that your official competitors utilize the online 2020-2021 practice competitions so they are familiar with the online competition platform.

The **2020-2021 MATHCOUNTS School Handbook** will be your primary resource for the Competition Series this year. It is designed to help your students prepare for each of the rounds of the test, plus build critical thinking and problem-solving skills. This section of the Guide for New Coaches will focus on how to use this resource effectively for your team.

COACHES: LOG IN AT MATHCOUNTS.ORG/COACHES TO GET THE PROBLEMS, ANSWERS, STEP-BY-STEP SOLUTIONS AND PROBLEM INDEX FOR PROBLEMS 201-250.

WHAT'S IN THE HANDBOOK? There is a lot included in the *School Handbook*, and you can find a full table of contents on pg. 9 of this book, but below are the sections that you'll use the most when coaching your students.

- **Handbook Problems:** 250 math problems (200 in the book and 50 online) divided into Warm-Ups, Workouts and

Stretches. These problems increase in difficulty as the students progress through the book. (pg. 12)

- **Solutions to Handbook Problems:** complete step-by-step explanations for how each problem can be solved. These detailed explanations are only available to registered coaches. (pg. 36)
- **Problem Index + Common Core State Standards Mapping:** catalog of all handbook problems organized by topic, difficulty rating and mapping to Common Core State Standards. (pg. 50)
- **Answers to Handbook Problems:** key available to the general public. Your students can access this key, but not the full solutions to the problems. (pg. 52)

There are 3 types of handbook problems to prepare students for each of the rounds of the competition. You'll want to have your students practice all of these types of problems.

Warm-Ups
14 Warm-Ups in handbook
10 questions per Warm-Up
no calculators used

Warm-Ups prepare students particularly for the Sprint Round.

Workouts
8 Workouts in handbook
10 questions per Workout
calculators used

Workouts prepare students particularly for the Target Round.

Stretches
3 Stretches in handbook
Number of questions and use of calculators vary by Stretch

Each Stretch covers a particular math topic that could be covered in any round. These help prepare students for all rounds.

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IS THERE A SCHEDULE I SHOULD FOLLOW FOR THE YEAR? On average coaches meet with their students for an hour once a week at the beginning of the year, and more often as the competitions approach. Practice sessions may be held virtually or in-person, before school, during lunch, after school, on weekends or at other times, coordinating with your school's schedule and avoiding conflicts with other activities.

Designing a schedule for your practices will help ensure you're able to cover more problems and prepare your students for competitions. We've designed the *School Handbook* with this in mind. Below is a suggested schedule for the program year that mixes in Warm-Ups, Workouts and Stretches from the *School Handbook*, plus free practice competitions from last year. This schedule allows your students to tackle more difficult problems as the Chapter Competition approaches.

Mid-August – September 2020 Warm-Ups 1, 2 + 3 Workouts 1 + 2	**October 2020** Warm-Ups 4 + 5 Workout 3 Mixture Stretch Practice Competition 1	**November 2020** Warm-Ups 6 + 7 Workout 4 Statistics Stretch Practice Competition 2	**December 2020** Warm-Ups 8 + 9 Workout 5 Pascal's Triangle Stretch Practice Competition 3
January 2021 Warm-Ups 10 +11 and Workout 6 Past Competitions (2020 School + Chapter) Practice Competition 4 *Select chapter competitors (required by this time)*		**February 2021** *2021 MATHCOUNTS Chapter Competition* *2021 MATHCOUNTS Chapter Invitational*	

FINISHED ALL THE PROBLEMS IN THE HARD-COPY HANDBOOK? FIND 50 MORE CHALLENGING PROBLEMS AT WWW.MATHCOUNTS.ORG/COACHES

You'll notice that in January you'll need to select the 1-15 student(s) who will represent your school in the Chapter Competition. This must be done by **January 15, 2021**. You'll identify your official competitors on the Art of Problem Solving (AoPS) Contest Platform.

It's possible you and your students will meet more frequently than once a week and need additional resources. If that happens, don't worry! You and your Mathletes can work together using the **Interactive MATHCOUNTS Platform**, powered by NextThought. This free online platform contains numerous *MATHCOUNTS School Handbooks* and past competitions, not to mention lots of features that make it easy for students to collaborate with each other and track their progress. You and your Mathletes can sign up for free at mathcounts.nextthought.com.

CHECK OUT THE INTERACTIVE MATHCOUNTS PLATFORM TO GET EVEN MORE HANDBOOK PROBLEMS + PAST COMPETITIONS!

And remember, just because you and your students will meet once a week doesn't mean your students can only prepare for MATHCOUNTS one day per week. Many coaches assign "homework" during the week so they can keep their students engaged in problem solving outside of team practices. Here's one example of what a 2-week span of practices in the middle of the program year could look like.

Monday	Tuesday	Wednesday (Weekly Team Practice)	Thursday	Friday
-Students continue to work individually on Workout 4, due Wednesday	-Students continue to work on Workout 4 -Coach emails team to assign new homework problem(s), due Wednesday	-Coach reviews solutions to Workout 4 -Coach gives Warm-Up 7 to students as timed practice and then reviews solutions -Students discuss solutions to homework problem(s) in groups	-Coach emails math team to assign Workout 5 as individual work, due Wednesday	-Students continue to work individually on Workout 5
-Students continue to work individually on Workout 5, due Wednesday	-Students continue to work on Workout 5 -Coach emails team to assign new homework problem(s), due Wednesday	-Coach reviews solutions to Workout 5 -Coach gives Warm-Up 8 to students as timed practice and then reviews solutions -Students discuss solutions to homework problem(s) in groups	-Coach emails math team to assign Workout 6 as group work, due Wednesday	-Students work together on Workout 6 using online Interactive Platform

WHAT SHOULD MY TEAM PRACTICES LOOK LIKE? Obviously every school, coach and group of students is different, and after a few practices you'll likely find out what works and what doesn't for your students. Here are some suggestions from veteran coaches about what makes for a productive practice.

- Encourage discussion of the problems so that students learn from each other
- Encourage a variety of methods for solving problems
- Have students write math problems for each other to solve
- Practice working in groups so students can learn from one another
- Practice oral presentations to reinforce understanding

On the following page is a sample agenda for a 1-hour practice session. There are many ways you can structure math team meetings and you will likely come up with an agenda that works better for you and your group. It also is probably a good idea to vary the structure of your meetings as the program year progresses.

MATHCOUNTS Team Practice Sample Agenda – 1 Hour

Review Assigned Homework Problem(s) (10 minutes)

- Have 1 student come to the board to show how s/he solved the problem(s).
- Have students divide into groups of 4 to discuss the solution presented and other methods for solving.
- Have 2 groups share an alternate solution.
- Discuss as a group other strategies to solve the problem (and help if any groups answered incorrectly).

Timed Practice with Warm-Up (15 minutes)

- Have students put away all calculators and have one student pass out Warm-Ups (face-down).
- Give students 12 minutes to complete as much of the Warm-Up as they can.
- After 12 minutes is up, have students hold up pencils and stop working.

Play Game to Review Warm-Up Answers (35 minutes)

- Have students divide into 5 groups (size will depend on number of students in meeting).
- Choose a group at random to start and then rotate clockwise to give each group a turn to answer a question. When it is a group's turn, ask the group one question from the Warm-Up.
- Have the group members consult their completed Warm-Ups and work with each other for a maximum of 60 seconds to choose the group's official answer.
- Award 2 points for a correct answer on questions 1-3, 3 points for questions 4-7 and 5 points for questions 8-10. The group gets 0 points if they answer incorrectly or do not answer in 60 seconds.
- Have all students check their Warm-Up answers as they play.
- Go over solutions to select Warm-Up problems that many students on the team got wrong.

GET MORE RESOURCES + ACTIVITIES TO MAKE TEAM MEETINGS FUN AT THE COACH SECTION OF THE MATHCOUNTS WEBSITE!

OK I'M READY TO START. HOW DO I GET STUDENTS TO JOIN? Here are some tips given to us from successful competition coaches and club leaders for getting students involved in the program at the beginning of the year.

- Ask Mathletes who have participated in the past to talk to other students about participating.
- Ask teachers, parent volunteers and counselors to help you recruit.
- Reach parents through school newsletters, PTA meetings or Back-to-School-Night presentations.
- Advertise around your school by:
    1. posting intriguing math questions (specific to your school) in your school and referring students to the first meeting for answers.
    2. designing a bulletin board or display case with your MATHCOUNTS poster (included in your School Competition Kit) and/or photos and awards from past years.
    3. attending meetings of other extracurricular clubs (such as honor society) so you can invite their members to participate.
    4. adding information about the MATHCOUNTS team to the website for your class or school.
    5. making a presentation at the first pep rally or student assembly.

Good luck in the competition! If you have any questions during the year, please contact the MATHCOUNTS national office at info@mathcounts.org.

COACH RESOURCES: WWW.MATHCOUNTS.ORG/COACHES

2020-2021 HANDBOOK MATERIALS

Thank you for being a coach in the MATHCOUNTS Competition Series this year!
We hope participating in the program is meaningful and enriching for you and your Mathletes.
Don't forget to log in at **www.mathcounts.org/coaches** for additional resources!

WHAT'S IN THIS YEAR'S HANDBOOK

COACHES:

FIND PROBLEMS, ANSWERS, SOLUTIONS + PROBLEM INDEX FOR #201-250 ONLINE AT WWW.MATHCOUNTS.ORG/COACHES

HIGHLIGHTED RESOURCES

ACCESS MORE RESOURCES AT WWW.MATHCOUNTS.ORG/COACHES

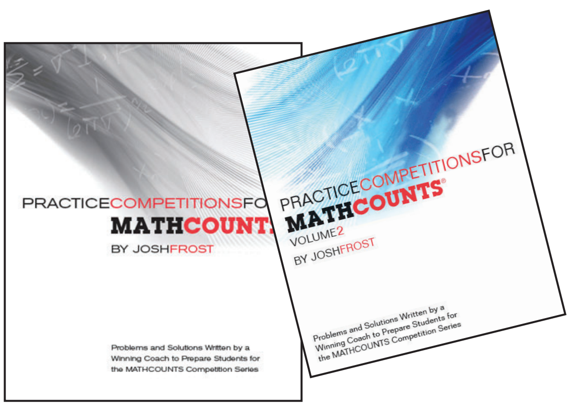
MATHCOUNTS OPLET

The Online Problem Library & Extraction Tool (OPLET) is a database of over 13,000 problems and over 5,000 step-by-step solutions. Create personalized quizzes, flash cards, worksheets and more!

SAVE $25 WHEN YOU BUY YOUR SUBSCRIPTION BY OCT. 1, 2020

www.mathcounts.org/myoplet

PRACTICE COMPETITIONS FOR MATHCOUNTS, VOL. 1 & 2

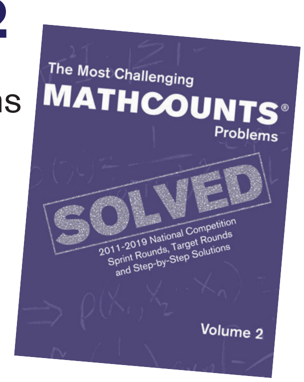


Practice books written by repeat national-level coach Josh Frost. Each volume includes 4 complete mock-competitions plus solutions.

www.mathcounts.org/store

MOST CHALLENGING MATHCOUNTS PROBLEMS SOLVED, VOL. 2

This book contains the Sprint Round and Target Round problems from the 2011 – 2019 MATHCOUNTS National Competitions plus never-before-available, step-by-step solutions!



COMING SEPTEMBER 2020!

Critical 2020-2021 Dates

Jun. 16 – Jan. 15	Submit your school's registration to participate in the Competition Series. The fastest way for schools to register is online at **www.mathcounts.org/compreg**. Schools also can download the MATHCOUNTS Competition Series Registration form and mail or email it with payment to: MATHCOUNTS Foundation – Competition Series Registrations 1420 King Street, Alexandria, VA 22314 *Email*: reg@mathcounts.org Parents who have confirmed their student's school is not participating in the program can register their child as a non-school competitor (NSC) this year. NSCs must register online and pay with a credit card at **www.mathcounts.org/nscreg**. Registered schools and NSCs will receive this year's School Competition Kit, which includes a hard copy of the *2020-2021 MATHCOUNTS School Handbook*. Kits are shipped on an ongoing basis between mid-August and January 31. **Questions?** Email the MATHCOUNTS national office at info@mathcounts.org.
Oct. 1 (postmark)	**Early Bird Registration Deadline** ($30/student for schools and $60/NSC) After October 1, registration increases to $35/student for schools and $65/NSC.
Oct. 15	**Practice Competition 1 Released**, available for registered schools and NSCs at the Art of Problem Solving (AoPS) Contest Platform through January 31, 2021.
Nov. 15	**Practice Competition 2 Released**, available for registered schools and NSCs at the AoPS Contest Platform through January 31, 2021.
Dec. 1 (postmark)	**Regular Registration Deadline** ($35/student for schools and $65/NSC) After December 1, registration will cost $40/student for schools and $70/NSC.
Dec. 15	**Practice Competition 3 Released**, available for registered schools and NSCs at the AoPS Contest Platform through January 31, 2021.
Early Jan.	Coaches should identify official chapter competitors on the AoPS Contest Platform at this time to ensure their students can practice using the competition platform. Identify official competitors by January 15 to ensure their participation in Practice Competition 4.
Jan. 15 (postmark)	**Final Day to Register, Pay and Add Students** ($40/student for schools and $70/NSC) MATHCOUNTS cannot guarantee participation for schools with outstanding invoices after January 15. Schools cannot add students to their registration after January 15.
Jan. 22-23	**Practice Competition 4**, available from 1:00pm ET on January 22 through 1:00pm ET on January 23 on the AoPS Contest Platform. Recommended for chapter competitors to use as a run-through for the Chapter Competition.
Feb. 5-6	**2021 Chapter Competition**, available from 1:00pm ET on February 5 through 1:00pm ET on February 6 on the AoPS Contest Platform.
Feb. 25	**2021 Chapter Invitational** at 7:00pm ET on February 25 on the AoPS Contest Platform. All competitors must take this competition at the same time.
Mar. 25	**2021 State Competition** at 7:00pm ET on March 25 on the AoPS Contest Platform. All competitors must take this competition at the same time.
May 8-11	**2021 Raytheon Technologies MATHCOUNTS National Competition** in Washington, DC

Mixture Stretch

1. ________ teaspoons Ming's recipe for sweet tea calls for 4 teaspoons of sugar. If Ming wants to make the tea 25% less sweet, how much less sugar should he use?

2. ________ Carla is mixing cherry, grape and lime candies in a bowl. Since her favorite flavor is cherry, she wants $\frac{2}{5}$ of the candies to be cherry. Since her least favorite flavor is lime, she wants $\frac{1}{4}$ of the candies to be lime. What fraction of the candies will be grape? Express your answer as a common fraction.

3. ________ A paving company makes concrete by adding water to a mix that is 1 part cement, 3 parts sand and 3 parts aggregate (stone). What fraction of this mix is aggregate? Express your answer as a common fraction.

4. ________ ounces A beef stew recipe calls for 12 ounces of beef, 4 ounces of carrots, 7 ounces of potatoes, 4 ounces of peas and 5 ounces of beef stock. Given that there are 16 ounces in a pound, how many ounces of potatoes are needed to make 4 pounds of this stew?

5. ________ % Jin adds 1 gallon of a water-and-bleach mixture that is 4% bleach to 2 gallons of a water-and-bleach mixture that is 10% bleach. What percent of the final mixture is bleach?

6. $ ________

Cashews cost $2.36 per pound, almonds cost $1.48 per pound and peanuts cost $0.98 per pound. To make a 20% profit, how much should Myrna charge per pound for a mixture that is 1 part cashews, 1 part almonds and 2 parts peanuts?

7. ________ gallons Manny's cleaning supply store receives a mixture of 80% detergent and 20% water in 15-gallon buckets. Manny would like a mixture of 60% detergent and 40% water in 5-gallon buckets. To make this, he combines some 80/20 mixture with some pure water in each 5-gallon bucket. How many gallons of pure water does Manny add to each 5-gallon bucket? Express your answer as a decimal to the nearest hundredth.

8. ________ buckets Based on the information in problem 7, how many 5-gallon buckets of 60/40 solution can Manny make from one 15-gallon bucket of 80/20 solution?

9. ________ quarts

Dara is mixing her own paint color, using 3 parts green paint to 2 parts blue to 1 part white. Given that there are 4 quarts in a gallon, if she needs 3 gallons of her paint, how many quarts of white paint should she buy?

10. ________ g/cm^3 To make a sand sculpture, Arthur used 2 cm^3 of red sand with a density of 4 g/cm^3, 7 cm^3 of yellow sand with a density of 5 g/cm^3 and 5 cm^3 of brown sand with a density of 6 g/cm^3. What is the average density of this sculpture in grams per cubic centimeter? Express your answer as a decimal to the nearest tenth.

Statistics Stretch

11.__________ A class of 28 students had a mean score of 72 on a math test. After the teacher realized that one of the questions had an alternative correct answer, he gave 4 points each to the 7 students who had given the alternative answer. What is the new mean test score?

12.__________ pages

stem	leaf
1	27 52 82
2	25 57 63 82 97
3	37 51 68 75

1|27 = 127 pages

The stem-and-leaf plot shows the number of pages in each book that Kalem read last summer. How many pages did Kalem read last summer?

13.__________ Based on the information in problem 12, what portion of the pages that Kalem read were in books having more than 275 pages? Express your answer as a common fraction.

14.__________ % A cafeteria offers apples, oranges and bananas with lunch. A student may take at most one of each fruit. Of the 61 students who got fruit with lunch, 5 students got only an apple and 7 got only an orange. Of the 16 students who got an apple and an orange, the 17 who got an orange and a banana, and the 20 who got an apple and a banana, 6 got all three fruits. What portion of the fruit taken by the 61 students were bananas? Express your answer to the nearest whole percent.

15.__________

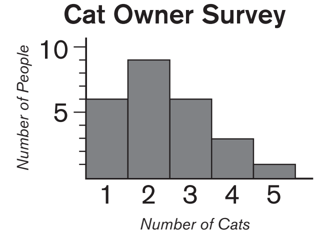
Chess Club Membership

	6th	7th	8th
Beginners	1	?	5
Advanced	2	?	6

The table shows the grade and skill level of members of the chess club. If half of the members are eighth graders and one-third of the beginners are seventh-graders, what fraction of chess club members are advanced chess players? Express your answer as a common fraction.

16.__________ If A, M and R represent the arithmetic mean, median and range of the set {13, 16, 18, 23, 25, 28, 30, 31}, what is the value of $A + M - R$?

Cat Owner Survey

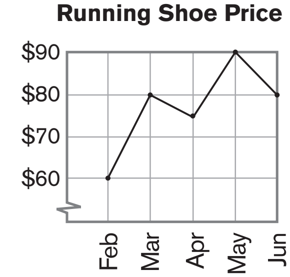


17.__________ cats This graph shows results of a survey of 25 cat owners. What is the mean number of cats per person surveyed? Express your answer as a decimal to the nearest hundredth.

18.__________ A total of 44 Mathletes competed in a MATHCOUNTS competition. The mean score for all the competitors was 28. The mean score for all competitors *except* the 16 highest scorers was 20. What was the mean score for the 16 highest scorers?

19.__________ % Bryce orders 6 bats, 60 baseballs and 8 gloves. If each bat costs \$29.95, a pack of 12 baseballs costs \$39.95 and a glove costs \$69.95, what portion of the total cost of this order is for the gloves? Express your answer to the nearest whole percent.

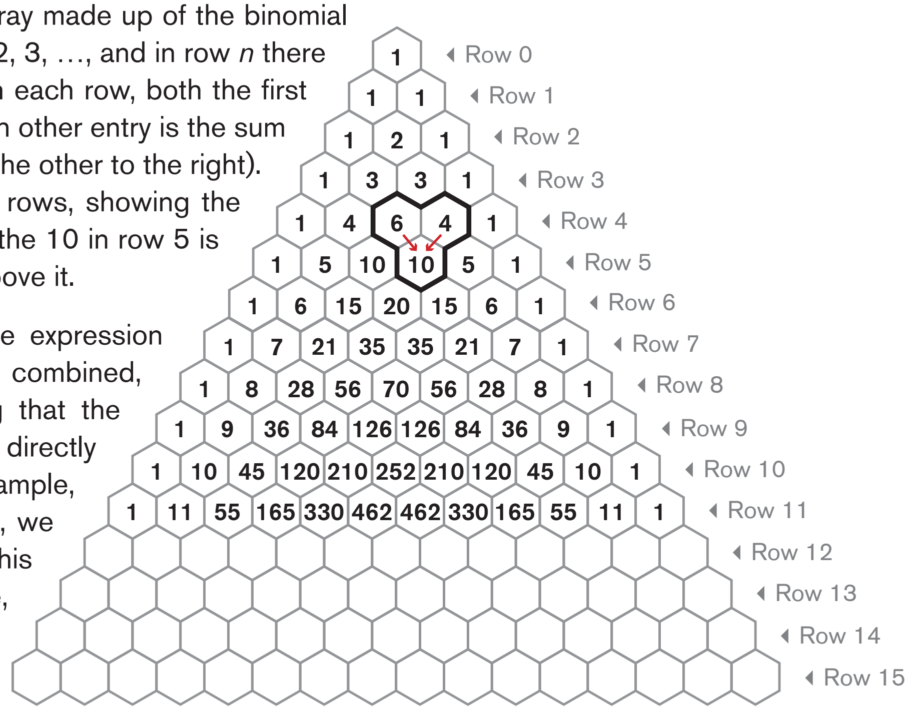
Running Shoe Price



20.__________ % The graph shows the price for a popular running shoe over five months. What is the absolute difference of the percent change in price from February to April and the percent change in price from April to June? Express your answer to the nearest tenth of a percent.

Pascal's Triangle Stretch

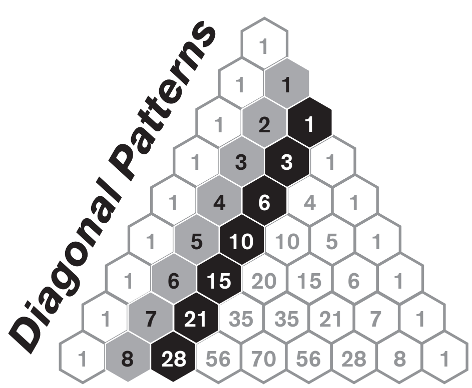
Pascal's triangle is a famous triangular array made up of the binomial coefficients. The rows are numbered 0, 1, 2, 3, …, and in row n there are $n + 1$ entries, numbered 0, 1, …, n. In each row, both the first entry (entry 0) and the last entry are 1. Each other entry is the sum of the two entries above it (one to the left, the other to the right). Here we have a Pascal's triangle with 16 rows, showing the entries for rows 0 through 11. Notice that the 10 in row 5 is the sum of the 6 and 4 in row 4, directly above it.



The binomial theorem says that when the expression $(x + y)^n$ is expanded and like terms are combined, the coefficient of $x^k y^{n-k}$ is $\binom{n}{k}$, meaning that the coefficients in this expansion can be read directly from row n in Pascal's triangle. For example, consider $(x + y)^4$. Expanding this binomial, we get $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$. In this case, $n = 4$. Referring to Pascal's triangle, we see that row 4 does indeed give us the coefficients of the terms in this expansion: 1, 4, 6, 4 and 1.

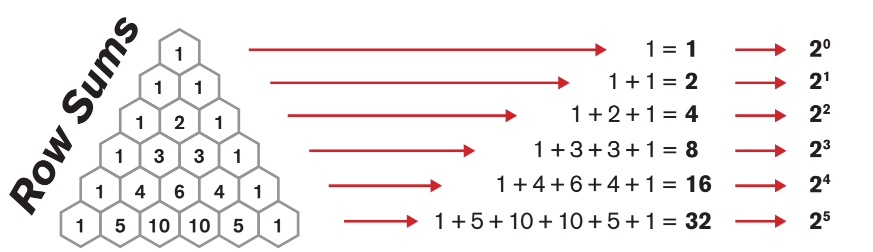
In general, the kth entry in row n is the binomial coefficient $\binom{n}{k} = {}_nC_k = \frac{n!}{k!(n-k)!}$, the number of combinations of n objects taken k at a time. Given five objects, to find the number of ways to choose three of them, or $\binom{5}{3}$, we locate entry 3 of row 5 in Pascal's triangle and see that there are **10** ways.

Below are a few interesting properties of Pascal's triangle.

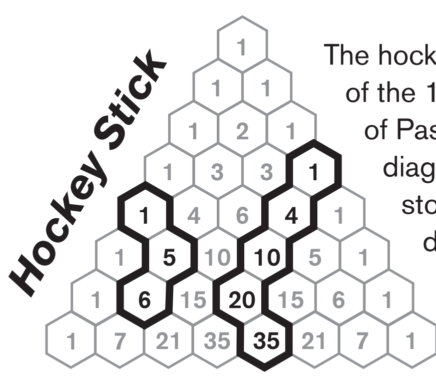


The gray diagonal contains the counting numbers: 1, 2, 3, 4, 5, ….

The black diagonal contains the triangular numbers: 1, 3, 6, 10, 15, …. In fact, entry 2 in row $(n + 1)$ is the nth triangular number.



The sum of the entries in the nth row of Pascal's triangle is equal to 2^n.



The hockey stick identity: Start at any of the 1s on the left or right side of Pascal's triangle. Sum entries diagonally in a straight line, and stop at any time. The next entry down diagonally in the opposite direction will equal that sum.

Solve the following problems, using what you've learned about Pascal's triangle. It may be helpful to fill in some of the missing entries in the Pascal's triangle on the previous page.

21. ________ What is the greatest entry in row 15 of Pascal's triangle?

22. ________ entries How many of the entries in row 14 of Pascal's triangle are even?

23. ________ What is the sum of the entries in row 12 of Pascal's triangle?

24. ________ choices A sports team of eight players must choose three starting players. How many different choices of three starters are there if the order in which they are chosen does not matter?

25. ________ What is the sum of the first 10 triangular numbers?

26. ________ When the expression $(x + 2)^8$ is expanded and like terms are combined, what is the coefficient of x^3?

27. ________ When the expression $(2x + y)^4$ is expanded and like terms are combined, what is the sum of the coefficients?

28. ________ When the expression $(x + 1)^{2022}$ is expanded and like terms are combined, the term with the greatest coefficient can be expressed as ax^b. What is the value of b?

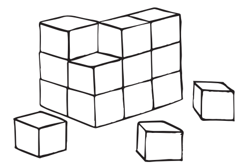
29. ________ A fair coin is flipped four times. What is the probability that it lands heads up at least as many times as it lands tails up? Express your answer as a common fraction.

30. ________ times Only the number 1 appears in Pascal's triangle more times than the number 3003 appears. How many times does 3003 appear in Pascal's triangle?

Warm-Up 1

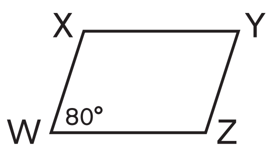
31. __________ What is the result when one hundred twenty-eight thousand is subtracted from one million?

32. __________ unit cubes How many unit cubes are required to create a larger cube with edge length 3 units?



33. __________ What is the remainder when the sum of the smallest and second-smallest prime numbers is divided by the third-smallest prime number?

34. __________ degrees



In parallelogram WXYZ, shown here, the measure of angle W is 80 degrees. What is the degree measure of angle X?

35. __________ What is the value of $6 \div 2 \times 3 + 8 \div 4 \times 2$?

36. __________ If Yasuko randomly selects a single-digit positive integer, what is the probability that it is *not* prime? Express your answer as a common fraction.

37. __________ What is the value of $0.001 \times \frac{1}{10} \times 10^5$?

38. __________ What is the value of $3(4x + 5y) - 2(7x - 3y)$ when $x = -2$ and $y = 3$?

39. __________ If $\frac{4}{18} = \frac{a}{27}$, what is the value of a?

40. __________ cups *Daifuku* is a snack made from glutinous rice flour and sweetened red bean paste. A recipe for 24 daifuku requires 3 cups of sugar. How many cups of sugar are needed to make 64 daifuku?

Warm-Up 2

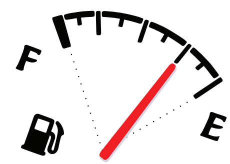
41. ______ : ______ p.m. Louisa leaves her house at 12:17 p.m., walks to the library, which takes 14 minutes, and remains there for 3 hours 27 minutes. If she walks home at the same speed as she walked to the library, at what time will she return home?

42. __________ What is the value of $7 - (3 - 4) + 11$?

43. __________ What is the least positive integer that is divisible by 4, 6 and 10?

44. ______ yards How many yards are in the perimeter of a square that measures 99 inches on each side?

45. ______ gallons When the Schwartzes left home, their gas gauge read $\frac{7}{8}$ full. When they reached their destination, their gauge read $\frac{1}{4}$ full. If their gas tank holds 16 gallons, how many gallons of gas did they use on their trip?



46. __________ What is the eighth term of the sequence that begins with 1, 3, 7, 13, 21, …?

47. __________ What is the value of x that satisfies the equation $5(x + 2) - 3(x - 8) = 16$?

48. ______ minutes

Andrew mowed one-half of a lawn, and Ben mowed one-third of the same lawn, each at a constant rate. If Andrew, continuing at the same rate, finished mowing the rest of the lawn in 12 minutes, how many minutes would it have taken him to mow the entire lawn by himself?

49. __________ If 125% of n is 30, what is 25% of n?

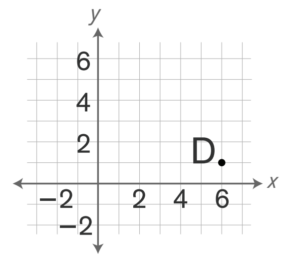
50. ______ outfits Don has four short-sleeved shirts, one each in black, white, red and gray, and two long-sleeved shirts, one each in red and gray. Don has three pairs of pants, one each in gray, black and tan. If Don chooses to wear only one black item or no black items at any one time, how many different outfits consisting of a shirt and a pair of pants can he make?

Warm-Up 3

51. ________ times The face of a clock has the numbers 1 through 12 painted on it. How many times is the digit 1 painted?

52. ________ What is the value of $5 + (-6) + 5 - (-6) + 5 - 6 + (-5 + 6)$?

53. ________ What is the sum of the coordinates of point D on the coordinate grid shown?



54. ________ What is the value of $\left(1 - \frac{1}{2}\right)^2\left(1 - \frac{1}{3}\right)^2$? Express your answer as a common fraction.

55. ________ What is the value of $0.123 + 1.032 + 2.301 + 3.210$? Express your answer as a decimal to the nearest thousandth.

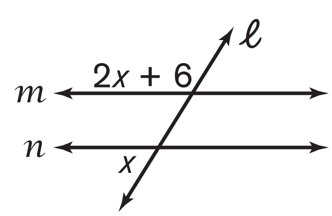
56. ________ primes Let $p(n)$ be the number of primes less than n. What is $p(50)$?

57. ________ cups

Maria wants to make a casserole to serve 12 people. She plans to use a recipe that calls for $1\frac{1}{4}$ cups of flour to serve 5 people. How many cups of flour will Maria need for her casserole?

58. ________ Let x and y represent the LCM and GCF, respectively, of 24 and 40. What is the value of $\frac{x}{y}$?

59. ________ degrees The figure shows parallel lines m and n with transversal ℓ. Based on the degree angle measures shown, what is the value of x?



60. ________ What is the greatest two-digit prime number that is one greater than a perfect square?

Warm-Up 4

61. ________ grams Each of 6 Mathletes ate a cheeseburger that contained 63 calories of fat. If there are 9 calories per gram of fat, how many total grams of fat did the Mathletes eat?

62. ________ What is the value of the 40th positive odd integer?

63. ________ units What is the distance between C(2, 1) and D(5, 5)?

64. ________

$1^2 = 1$

$11^2 = 121$

$111^2 = 12{,}321$

$1111^2 = 1{,}234{,}321$

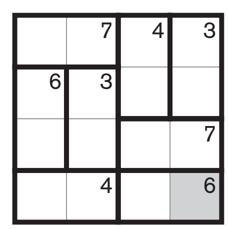
Based on the pattern shown, what is the sum of the digits when $11{,}111^2$ is calculated?

65. ________ What is the value of $1 + 2 \times 3 - 4 + 5 \times 6 - 7 + 8 \times 9$?

66. ________ % Last school year, the Math Club had 20 members. This school year, there are 28 members. By what percent did the membership of the Math Club increase?

67. ________ in² What is the area of a rectangle that has width $2\frac{3}{4}$ inches and length $3\frac{2}{5}$ inches? Express your answer as a mixed number.

68. ________ When the grid shown is filled in correctly, each of the numbers 1 through 4 will appear exactly once in each row and column. The small number in one corner of a heavily outlined rectangle is the sum of the numbers that belong in that rectangle. What number must be in the shaded cell of the grid?



69. ________ What is the value of $\dfrac{1+\frac{2}{3}}{2-\frac{3}{4}}$? Express your answer as a common fraction.

70. ________ Let x ☼ $y = \dfrac{(x+8)^2}{y}$. What is the value of 2 ☼ (2 ☼ 4)?

Warm-Up 5

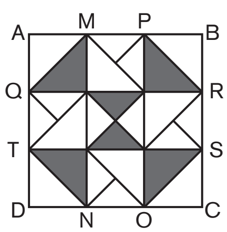
71. ______ points In the National Sports League (NSL), a team earns 3 points for a regulation win, 2 points for an overtime win and 1 point for an overtime loss. How many total points did an NSL team with 29 regulation wins, 10 overtime wins and 4 overtime losses earn?

72. ______ What common fraction is equivalent to 48.55 − 47.37?

73. ______ If s equals the square root of the reciprocal of 1.21, what is the value of s? Express your answer as a common fraction.

74. ______ points A group of 4 students took a math test. The mean of the numbers of points scored by the students is 95 points out of a possible 100 points. What is the minimum possible number of points scored by any student?

75. ______



In square ABCD, shown here, segments MN and OP trisect sides AB and DC, and segments QR and ST trisect sides AD and BC. What is the ratio of the combined area of the shaded regions to the area of square ABCD? Express your answer as a common fraction.

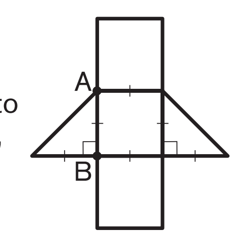
76. ______ All the students at the playground are wearing either long pants or shorts, and some are wearing a hat. Here is a table where Mossi recorded how many students are wearing various items. What is the probability that a randomly selected student on this playground is wearing shorts but no hat? Express your answer as a common fraction.

	Hat	No Hat
Pants	𝍸 |	||||
Shorts	𝍸	𝍸

77. ______ What is the result when −8 × (−4) − (−8) is divided by the sum −6 + (−4)?

78. $ ______ Simon and Theo paid a total of $15 for lunch. Simon paid $\frac{2}{3}$ the amount that Theo paid. How many more dollars than Simon did Theo pay?

79. ______ ft^3 Molly is folding a right triangular prism out of a piece of poster board cut into the shape shown. If the distance between point A and point B is 24 inches, what will the volume of the prism be in cubic feet?

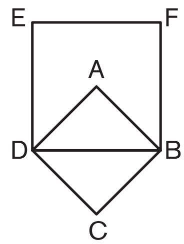


80. ______ What is the value of 202,020,202,020 × 2021 − 202,120,212,021 × 2020?

Warm-Up 6

81. __________ If $A = 2^3 - 3^2 + 4(5 + 1)$ and $B = 7^2 - 2(3 + 1)^2$, what is the value of $A - B$?

82. __________



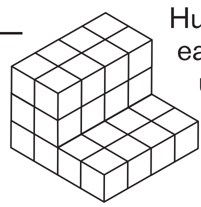
What is the ratio of the area of square ABCD to that of square BDEF in the figure shown? Express your answer as a common fraction.

83. __________ What is the median of the first ten prime numbers?

84. __________ Let the value of a word equal the sum of the values of its letters. Suppose MATH has value M + A + T + H = 85, and suppose M = 8, A = 3H, T = 73 and H = 1. If S = 3M − 15 and N = 2H + S, what is the value of SAMANTHA?

85. __________ What is the remainder when the sum $10^3 + 4^2$ is divided by 8?

86. __________



Hugo has a solid like the one shown, composed of 32 unit cubes. He paints each face of this solid red and then separates the solid into the 32 individual unit cubes. What is the probability that a randomly selected unit cube has exactly one painted face? Express your answer as a common fraction.

87. __________ If x equals twice its reciprocal, what is the value of x^4?

88. ________ rooms A robotic vacuum cleaner vacuums one hotel room in $\frac{3}{10}$ hour. At this rate, how many rooms of the same size will it vacuum in 3 hours?

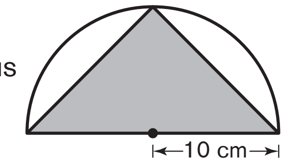
89. __________ Nya starts with an integer N and repeatedly subtracts 6. Mya starts with an integer M and repeatedly adds 8. When Nya and Mya have each performed their respective repeated operation 13 times, both have a resulting value of 25. What is the value of $N + M$?

90. __________ What is the value of the product $\left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{3}\right)\left(1 + \frac{1}{4}\right) \cdots \left(1 + \frac{1}{19}\right)$?

Warm-Up 7

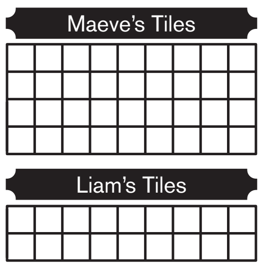
91. __________ students This school term, 25% of the students in Ms. Norton's class earned a final grade of A. If 7 students earned an A this term, how many students are in Ms. Norton's class?

92. __________ cm^2 The figure shows an isosceles triangle inscribed in a semicircle of radius 10 cm. What is the area of the triangle?



93. __________ What is the integer value of $\dfrac{1.4\times10^5}{7\times10^2}$?

94. __________ tiles

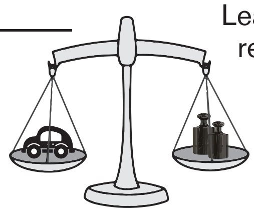


Maeve creates a design by painting $\frac{1}{3}$ of her square tiles. Liam creates a design by painting $\frac{5}{9}$ of his square tiles. What is the combined number of painted tiles in Maeve's and Liam's designs?

95. __________ If $\dfrac{a}{b} = \dfrac{2}{3}$ and $a + b = 100$, what is the value of b?

96. __________ The mean, median and unique mode of a list of five positive integers are all equal to 5. What is the greatest possible value of an integer in this list?

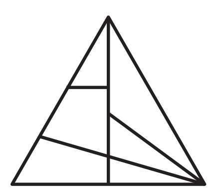
97. __________



Leah puts a toy car weighing 6 ounces on the left side of a balance. Then, reaching into a bag that contains four weights, measuring 1, 2, 4 and 5 ounces, she randomly removes two weights, without replacement. If she places the two weights on the right side of the balance, what is the probability that the balance levels? Express your answer as a common fraction.

98. __________ What is the value of $\dfrac{7^4-3^4}{7^2+3^2}$?

99. __________ triangles How many triangles of any size are in the figure shown?

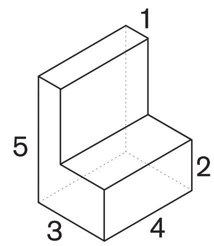


100. __________ arrangements Sara and Ben make a playlist for a road trip. Each chooses 5 songs for the playlist, and they order the songs so that no two consecutive songs were added to the list by the same person. How many such song arrangements are possible for their playlist, assuming no song is repeated?

Warm-Up 8

101. __________ What number is one-third of two-fifths of 90?

102. __________ units³



What is the volume of the figure shown, in which all adjacent edges are perpendicular?

103. __________ When a number n is divided by -6 and the quotient is increased by 6, the result is 3. What is the value of n?

104. __________ What common fraction is equivalent to $1 + \cfrac{1}{1+\cfrac{1}{1+\frac{1}{3}}}$?

105. __________ What is the absolute difference between the slope of the line passing through the points given in this table and the slope of the line given by $2x - y = 4$? Express your answer as a common fraction.

x	y
1	2
4	7
7	12

106. __________ What is the value of $2^5 \times 4^{-2}$?

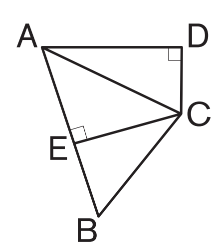
107. __________ years old

Armin's 13th birthday was on Saturday, July 4, 2020. How old will Armin be when his birthday next falls on a Saturday?

108. __________ What is the value of a in the geometric sequence $-1, 3, -9, a, -81$?

109. __________ Suppose that N is an integer such that 3^N is a factor of 10!. What is the greatest possible value of N?

110. __________ units In the figure shown, CD = 6 units, $m\angle CAD = 30°$, $m\angle ACE = 45°$ and $m\angle ABC = 60°$. What is the length of segment EB? Express your answer in simplest radical form.



Warm-Up 9

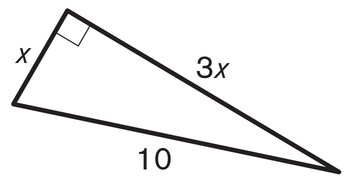
111. ______

A bag contains 12 hair bows: 5 red, 4 white and 3 blue. Jo Jo reaches into the bag and randomly pulls out two bows without replacement. What is the probability that those two bows are the same color? Express your answer as a common fraction.

112. ______ If $4\left(\frac{5}{8}x - \frac{1}{2}\right) = 3$, what is the value of x?

113. ______ A fair coin is flipped 5 times. What is the probability that no two consecutive flips have the same result? Express your answer as a common fraction.

114. ______ A right triangle has side lengths x, $3x$ and 10, as shown. What is the value of x? Express your answer in simplest radical form.



115. ______ What is the value of $4^{12} \times 125^7$? Express your answer in scientific notation.

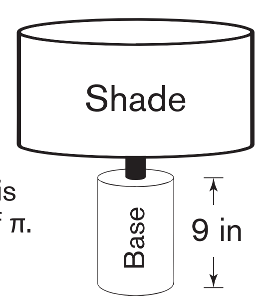
116. ______ cm Rachelle draws a rectangle of perimeter 46 cm and area 90 cm^2. Evan draws a rectangle with twice the perimeter and half the area of Rachelle's rectangle. What is the smaller dimension of Evan's rectangle?

117. ______ coins Tonya found \$2.25 in nickels and quarters in her sofa cushions. If the number of nickels Tonya found is five more than three times the number of quarters she found, what is the total number of coins Tonya found?

118. ______ A line passes through the points $(-7, 1)$, $(5, 7)$ and $(0, b)$. What is the value of b? Express your answer as a common fraction.

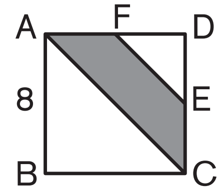
119. ______ Every positive integer can be expressed in the form $6n + k$, where $0 \le k \le 5$. If 1841 is expressed in this form, what is the value of $n + k$?

120. ______ in^3 A lamp's base and shade are both cylindrical as shown. The shade has circumference 18π inches, which is three times that of the lamp's base. If the lamp's base is made of solid brass and has height 9 inches, what is the volume of brass in the lamp's base? Express your answer in terms of π.



Warm-Up 10

121.________ cm^2 Square ABCD, shown here, has side length 8 cm. If E and F are the midpoints of sides CD and AD, respectively, what is the area of shaded trapezoid ACEF?



122.________ If $f(x) = 2x^3 - 5x^2 + 9x + 4$, what is the value of $f(-2)$?

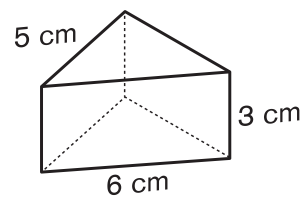
123.________ Three numbers are selected at random without replacement from the set {2, 3, 5, 7, 11, 13}. What is the probability that the sum of the three numbers will be a multiple of 3? Express your answer as a common fraction.

124.________

In the square shown, stripes run parallel to the sides and divide the top and right sides of the square into congruent segments. What fraction of the figure is shaded? Express your answer as a common fraction.

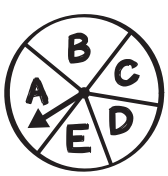
125.________ What is the value of $\left[\left(\frac{2}{3}\right)^2 \div 4\right]^{\frac{1}{2}}$?

126.________ cm^2 The right triangular prism shown has bases that are isosceles triangles. If each triangular base has congruent sides of length 5 cm and a third side of length 6 cm, and the prism has height 3 cm, what is the surface area of this prism?



127.________ What is the smallest five-digit number that has exactly one 4 and exactly one 6 and is a multiple of 9?

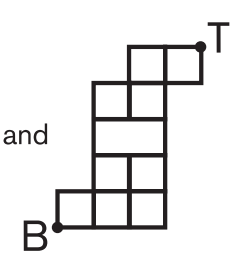
128.________



In the circular spinner shown, sections A and B are congruent, each with a 90-degree central angle, and sections C, D and E are all congruent. When Allie spins this spinner, what is the probability that it lands on section A or D? Express your answer as a common fraction.

129.________ Two numbers have a geometric mean of 4 and an arithmetic mean of 5. What is the larger of the two numbers?

130.________ paths In the figure shown, how many paths from B to T are there that move up and right along the line segments?



Warm-Up 11

131.__________ If $f(x) = x^2$ and $g(x) = 2x - 1$, what is the value of $f(5) - g(8)$?

132.__________ A *nugget number* is a positive integer that can be obtained by adding together any combination of the numbers 6, 9 and 20. For example, 75 is a nugget number because $20 + 20 + 20 + 9 + 6 = 75$, whereas, 34 is not a nugget number. What is the largest nugget number less than 200?

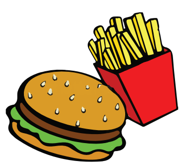
133.__________ What is the value of $81^{\frac{3}{4}}$?

134.__________ arrangements How many unique arrangements are there of all six letters of SQUARE?

135.__________ ways In how many ways can all the numbers 1, 2, 3, 4, 5, 6 and 7 be separated into two groups, so that the sum of the numbers in both groups is the same?

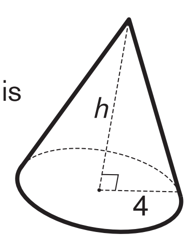
136.__________ If $x + \frac{1}{x} = -2$, what is the value of $x^4 + \frac{1}{x^4}$?

137.__________ burgers



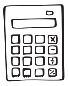
Bodacious Burgers sells burgers for \$2.50 each and french fries for \$0.99 per container. Four friends ate at Bodacious Burgers. The bill for the meal was \$17.97 before tax. How many burgers were ordered?

138.__________ cm A right circular cone with base radius 4 cm has surface area 56π cm^2. What is the height of the cone? Express your answer in simplest radical form.



139.__________ What is the least positive two-digit integer that leaves a remainder of 3 when divided by each of the numbers 4, 5 and 6?

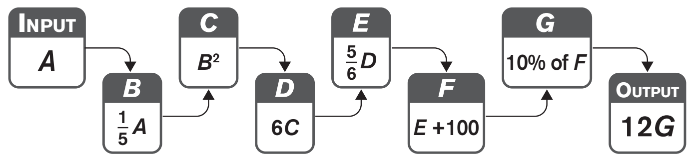
140.__________ units A triangle has sides of lengths 18, 24 and 30 units. What is the length of the shortest altitude of this triangle? Express your answer as a common fraction.



Workout 1

141.__________ What is the sum of the integers less than 100 that have both 6 and 9 as divisors?

142.__________ Devon wrote a program that takes a positive integer *A* as an input and performs a series of operations, each time assigning the result to a new variable, as shown. If the output of the program is 144, what was the value of the input *A*?



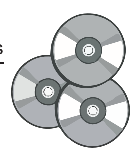
143.__________ What is the arithmetic mean of the numbers 3, 66 and 999?

144.__________ hours Grace notices interstate highway markers placed at the end of each mile and numbered consecutively: 1, 2, 3, 4, …. If an accurate speedometer says Grace is traveling 72 mi/h, how many hours will it take her to travel from mile marker 7 to mile marker 29? Express your answer as a decimal to the nearest hundredth.

145.__________ % Cliff's piano has 52 white keys and 36 black keys. What percent of the keys on his piano are white? Express your answer to the nearest whole percent.

146.__________ mL On average, in men, 39 mL per kilogram of body weight is blood plasma. In women, 40 mL per kilogram of body weight is blood plasma. Rob weighs 80 kg, and Kristen weighs 60 kg. How many more milliliters of blood plasma does Rob have than Kristen?

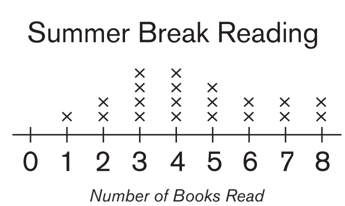
147.__________ times



A CD spins 5 times per second. How many times will the CD spin while playing a song that is 3 minutes 43 seconds long?

148.__________ What is the greatest possible product of two positive integers whose sum is 34?

149.__________ books The line plot shows the number of books that each student in Ms. Coleman's homeroom reported reading over summer break. What was the mean number of books read by a student in this homeroom? Express your answer as a decimal to the nearest tenth.



150.__________ degrees Angles A and B are complementary, with $m\angle A = 5x - 6$ degrees and $m\angle B = 3x$ degrees. What is the degree measure of angle A?

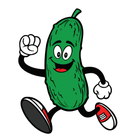
Workout 2

151. ________ feet If 1 inch is equivalent to 2.54 cm, how many feet are in 50 cm? Express your answer as a decimal to the nearest hundredth.

152. ________ integers How many two-digit positive integers are there with the property that the sum of the integer's digits equals the product of those digits?

153. $ ________ Alistair has 40 coins in his pocket, including at least one penny, one nickel, one dime and one quarter. If he has no other type of coin in his pocket, what is the greatest possible total value of the coins in Alistair's pocket?

154. ________ % If Sanjay runs 80% as far as Jerome in the eighth-grade pickle-rolling race, what percent of Sanjay's distance does Jerome run?



155. ________ What is the least common multiple of the first five positive cubes?

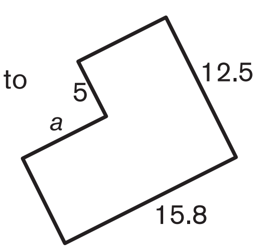
156. ________ units² What is the area of the triangle bounded by the line $3x + 2y = 12$ and the x- and y-axes?

157. ________ hours The table shows the hours that Gene, Doug and Pat worked canning corn one Saturday in July. What is the combined number of hours they worked canning corn? Express your answer as a decimal to the nearest hundredth.

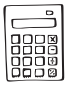
	1st Shift		2nd Shift	
	Start	End	Start	End
GENE	7:00 a.m.	10:15 a.m.	10:30 a.m.	12:30 p.m.
DOUG	7:30 a.m.	10:15 a.m.	10:30 a.m.	12:30 p.m.
PAT	7:15 a.m.	10:15 a.m.	10:30 a.m.	11:45 a.m.

158. ________ What is the median of the integers between 1 and 1000 that are divisible by 28?

159. ________ units The figure shows a hexagon in which adjacent sides are perpendicular to each other. The hexagon with the given side lengths has an area of 160 units^2. What is the value of a? Express your answer as a decimal to the nearest tenth.



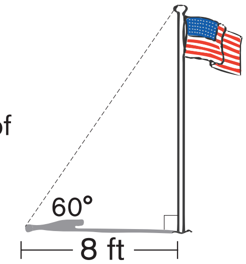
160. ________ primes Let an *optimus prime* be a prime number whose digit sum is also a prime number. How many of the first 10 primes are optimus primes?



Workout 3

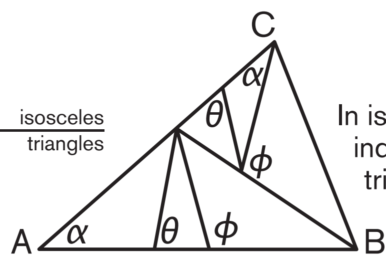
161.__________ What is the median of the set $\left\{\frac{7}{3}, 2, 1.5, 1\frac{1}{4}\right\}$? Express your answer as a decimal to the nearest hundredth.

162.________ years old Marco's age is between 20 and 60. The square of his age is a four-digit number, and the sum of the digits of his age is 8. How old is Marco if his age has two different digits and is not prime?



163.________ feet The flagpole shown is perpendicular to the ground and casts a shadow of 8 feet. The angle of elevation to the top of the pole from the end of the shadow is 60 degrees. How tall is the pole? Express your answer as a decimal to the nearest tenth.

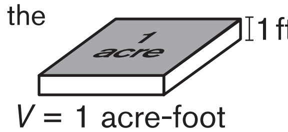
164.________ blueberries The number of blueberries left in a bowl is reduced by half every 2 hours. Sam filled the bowl with blueberries at 9:00 a.m. When he checked the bowl at 7:00 p.m., there were 5 blueberries left. How many blueberries did Sam originally place in the bowl at 9:00 a.m.?



165.________ isosceles triangles In isosceles triangle ABC, shown here, AB = AC. For the angles indicated, $\alpha = 36°$, $\theta = 72°$ and $\phi = 108°$. How many isosceles triangles of any size are in this figure?

166.________ integers How many integers between 1,000,000 and 2,000,000 are divisible by 99?

167.________ gallons An acre-foot is defined as the volume of a rectangular prism with a base area of 1 acre and a depth of 1 foot, as shown. Given that 1 acre is defined as the area of a 66-foot × 660-foot rectangle and that 1 gallon equals 231 in^3, how many gallons are in 1 acre-foot of water? Express your answer as an integer to the nearest thousand.

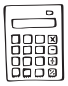


168.________ % What percent of positive integers less than or equal to 100 are divisible by 3?

169.________ students Ali's middle school has a total of 300 students on Team A and Team B. After 30 students are moved from Team A to Team B, there are twice as many students on Team A as there are on Team B. How many students were originally on Team B?

170.________ cats The mean number of cats living in each of the 50 apartments in a particular apartment building is 0.44 cats. A total of 32 apartments in the building are cat-free. What is the mean number of cats in the apartments that have at least one cat? Express your answer to the nearest tenth.

Workout 4



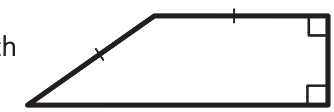
171. ________ What common fraction t satisfies the equation $\frac{t}{3t+1} = \frac{4}{5}$?

172. ________ ways

Mr. Scott has five algebra books and four geometry books. He wants to arrange them all on a single shelf. If Mr. Scott keeps all of the algebra books together and all of the geometry books together, how many ways can he arrange these books on his shelf?

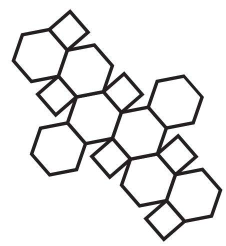
173. ________ If $A + B = C + 1$, $B + C = D - 1$, $C + D = E + 1$, $D + E = F - 1$, $E + F = G + 1$, $F + G = A - 1$ and $G + A = B + 1$, what is the value of $A + B + C + D + E + F + G$?

174. ________ cm^2 What is the area of the trapezoid shown, with top base of length 10 cm and sides of lengths 10 cm and 6 cm?



175. ________ tests The mean of Danielle's test scores is 85. If Danielle's lowest test score, which is 61, were to be discarded, the mean of her remaining test scores would be 88. How many tests did Danielle take?

176. ________ vertices This figure shows the net of a three-dimensional shape called a truncated octahedron. How many vertices does a truncated octahedron have?

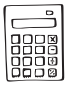


177. ________ cups Jamie is making pudding, using a recipe that calls for 1.5 cups of milk and 2 cups of flour. He has 7.75 cups of milk and would like to make a batch of pudding using all of the milk. How many cups of flour will he need in order to keep the ratio of ingredients constant? Express your answer as a decimal to the nearest tenth.

178. ________ years old When Shawna turned 21 years old, she was three times as old as Shelby. How old will Shawna be when she is twice as old as Shelby?

179. ________ The sum of a number x and its reciprocal equals $-\frac{17}{4}$. What is the sum of all possible values of x? Express your answer as a common fraction.

180. ________ children There were 9 adults and 11 children at the movie at 11:45 a.m. By 11:50 a.m., 7 more adults and 8 more children were at the movie. At 12:00 p.m., there were 60 adults and children, and the ratio of adults to children was the same as at 11:45 a.m. How many more children came to the movie between 11:50 a.m. and 12:00 p.m.?



Workout 5

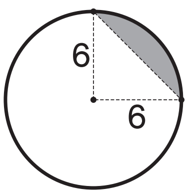
181.__________ If each of the numbers 2, 7, 3 and 8 is assigned to one of the variables *a*, *b*, *c* and *d*, what is the greatest possible value of $ab + bc + cd$?

182.__________ cm If the volume of the air in an exercise ball is 137,250 cm^3, what is the inside diameter of the ball? Express your answer to the nearest whole number.

183.__________ A square array of numbers is called an *arithmetic square* if each row and column forms an arithmetic progression. If the remaining entries in the array shown are filled in to create an arithmetic square, what is the greatest number in the array?

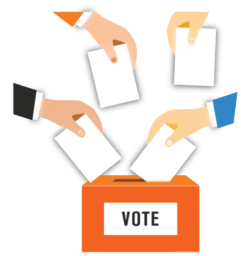
			8	
		36		
			36	
	66			

184.__________ units^2



A right triangle is inside a circle, with one vertex of the triangle at the center of the circle and the other two vertices on the circle, as shown. If the circle has radius 6 units, what is the area of the shaded segment? Express your answer as a decimal to the nearest tenth.

185.__________ votes Students at Baldwin Middle School cast 432 votes for student government president. If the number of votes the losing candidate received was 60% of the number that the winner received, how many more votes than the loser did the winner receive?



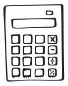
186. $__________ Michelle teaches on Tuesday and Thursday evenings. On Tuesdays she makes $112.50, and on Thursdays she makes $135. There are 31 days this month. What is the minimum amount that Michelle will earn this month?

187.__________ ways Including the example shown, how many ways are there to completely cover this 2 × 7 grid with nonoverlapping 1 × 1 and 2 × 2 tiles, if rotations and reflections of these arrangements are considered distinct?

188.__________ If $S_1 = 4$ and $S_n = S_{n-1} + (3 + n)^n$, what is the value of S_3?

189.__________ The point P(7, 4) is reflected across the line ℓ to the point P′(2, 6). What is the slope of line ℓ? Express your answer as a common fraction.

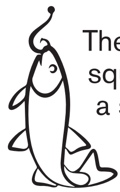
190.__________ drops A veterinarian needs to prepare a vitamin mixture for parakeets. The ratio is 1 drop of vitamin oil to 75 drops of water. How many drops of vitamin oil are needed to prepare 1520 drops of the mixture?



Workout 6

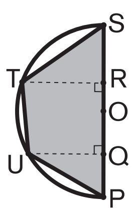
191. ________ units^2 The triangle with vertices A(0, 4), B(3, 0) and C(4, 7) is dilated by a factor of $\sqrt{2}$ about the origin. What is the area of the dilated triangle?

192. ________ prizes A raffle has prizes valued at \$1, \$3, \$15, \$60, \$120, \$360 and \$1800. If exactly \$10,000 in prizes is to be awarded, what is the least number of prizes that can be awarded, assuming that there will be at least one prize of each value awarded?

193. ________ pounds  The estimated weight of a salmon, in pounds, is the product of its length and the square of its girth, both in inches, divided by 775. What is the estimated weight of a salmon with length 28.5 inches and girth 10.25 inches? Express your answer as a decimal to the nearest tenth.

194. ________ Point Q is the reflection of P(a, 4) over the x-axis, and R is the reflection of Q over the y-axis. If the area of $\triangle PQR$ is 12 units^2 and $a > 0$, what is the value of a? Express your answer as a decimal to the nearest tenth.

195. ________ hours Trains pass through a rail yard on five separate tracks at five different frequencies: every 50 minutes, every hour, every 90 minutes, every 2 hours and every 3 hours. If trains pass through on all five tracks at 9:00 a.m., how many hours will elapse before trains next pass through on all five tracks at the same time?

196. ________ units^2 

Quadrilateral PSTU is inscribed in semicircle O, as shown, with PQ = 3 units, QR = 5 units and RS = 4 units. What is the area of quadrilateral PSTU? Express your answer as a decimal to the nearest tenth.

197. ________ units Points A, B, C, D and E on a line are positioned so that B is the midpoint of segment AD, D is three-fourths of the way from A to E, and C is one-fifth of the way from B to E. If AB = 6 units, what is the value of CE?

198. ________ % Vick pays \$50 for 33 pounds of cashews, which he separates into smaller $\frac{3}{8}$-pound bags. If Vick then sells each bag for \$0.79, what percent of this price is profit? Express your answer to the nearest tenth of a percent.

199. \$________ To make bracelets, Alice bought 12 red beads and 10 white beads for \$14, while Nevea bought 8 red beads and 15 white beads for \$13.50. What is the cost of a red bead?

200. ________ If $8 - x^2 - y^2 - 2xy = 0$, for real numbers x and y, what is the value of $|x + y|$? Express your answer in simplest radical form.

Competition Coach Toolkit

This is a collection of lists, formulas and terms that Mathletes frequently use to solve problems like those found in this handbook. There are many others we could have included, but we hope you find this collection useful.

Fraction	Decimal	Percent
1/2	0.5	50
1/3	$0.\overline{3}$	$33.\overline{3}$
1/4	0.25	25
1/5	0.2	20
1/6	$0.1\overline{6}$	$16.\overline{6}$
1/8	0.125	12.5
1/9	$0.\overline{1}$	$11.\overline{1}$
1/10	0.1	10
1/11	$0.\overline{09}$	$9.\overline{09}$
1/12	$0.08\overline{3}$	$8.\overline{3}$

Common Arithmetic Series

$$1 + 2 + 3 + 4 + \cdots + n = \frac{n(n+1)}{2}$$

$$1 + 3 + 5 + 7 + \cdots + (2n - 1) = n^2$$

$$2 + 4 + 6 + 8 + \cdots + 2n = n^2 + n$$

Prime Numbers

2	43
3	47
5	53
7	59
11	61
13	67
17	71
19	73
23	79
29	83
31	89
37	97
41	

Combinations & Permutations

$${}_nC_r = \frac{n!}{r!(n-r)!} \qquad {}_nP_r = \frac{n!}{(n-r)!}$$

n	n^2	n^3
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729
10	100	1000
11	121	1331
12	144	1728
13	169	2197
14	196	2744
15	225	3375

Divisibility Rules

- 2: units digit is 0, 2, 4, 6 or 8
- 3: sum of digits is divisible by 3
- 4: two-digit number formed by tens and units digits is divisible by 4
- 5: units digit is 0 or 5
- 6: number is divisible by both 2 and 3
- 8: three-digit number formed by hundreds, tens and units digits is divisible by 8
- 9: sum of digits is divisible by 9
- 10: units digit is 0

Geometric Mean

$$\frac{a}{x} = \frac{x}{b} \quad \text{and} \quad x = \sqrt{ab}$$

Equation of a Line

Standard Form

$$\boxed{Ax + By = C}$$

Slope-Intercept Form

$$\boxed{y = mx + b}$$

m = slope $\qquad b$ = y-intercept

Point-Slope Form

$$\boxed{y - y_1 = m(x - x_1)}$$

m = slope $\qquad (x_1, y_1)$ = point on the line

Distance Traveled

Distance = Rate × Time

Quadratic Formula

For $ax^2 + bx + c = 0$, where $a \neq 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Pythagorean Triples

(3, 4, 5)	(5, 12, 13)	(7, 24, 25)
(8, 15, 17)	(9, 40, 41)	(12, 35, 37)

Difference of Squares

$$a^2 - b^2 = (a + b)(a - b)$$

Sum and Difference of Cubes

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Circles

Circumference	Area
$2 \times \pi \times r = \pi \times d$	$\pi \times r^2$

For radius r

Arc Length	Sector Area
$\frac{x}{360} \times 2 \times \pi \times r$	$\frac{x}{360} \times \pi \times r^2$

For central angle of x degrees

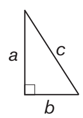
Given $A(x_1, y_1)$ and $B(x_2, y_2)$

Distance from A to B = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint of $\overline{AB}$ = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

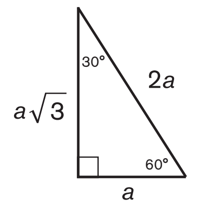
Slope of $\overline{AB}$ = $\frac{y_2 - y_1}{x_2 - x_1}$

Pythagorean Theorem

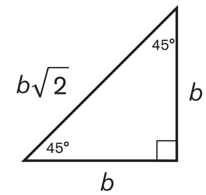


$a^2 + b^2 = c^2$

Special Right Triangles



30-60-90 Right Triangle



45-45-90 Right Triangle

Area of Polygons

Square	side length s	s^2
Rectangle	length l, width w	$l \times w$
Parallelogram	base b, height h	$b \times h$
Trapezoid	bases b_1, b_2, height h	$\frac{1}{2}(b_1 + b_2) \times h$
Rhombus	diagonals d_1, d_2	$\frac{1}{2} \times d_1 \times d_2$
Triangle	base b, height h	$\frac{1}{2} \times b \times h$
Triangle	semi-perimeter s, side lengths a, b, c	$\sqrt{s(s-a)(s-b)(s-c)}$
Equilateral Triangle	side length s	$\frac{s^2\sqrt{3}}{4}$

Polygon Angles (*n* sides)

Sum of the interior angle measures:

$$180 \times (n - 2)$$

Central angle measure of a regular polygon:

$$\frac{360}{n}$$

Interior angle measure of a regular polygon:

$$\frac{180 \times (n - 2)}{n} \quad \text{or} \quad 180 - \frac{360}{n}$$

Solid	Dimensions	Surface Area	Volume
Cube	side length s	$6 \times s^2$	s^3
Rectangular Prism	length l, width w, height h	$2 \times (l \times w + w \times h + l \times h)$	$l \times w \times h$
Cylinder	circular base radius r, height h	$2 \times \pi \times r \times h + 2 \times \pi \times r^2$	$\pi \times r^2 \times h$
Cone	circular base radius r, height h	$\pi \times r^2 + \pi \times r \times \sqrt{r^2 + h^2}$	$\frac{1}{3} \times \pi \times r^2 \times h$
Sphere	radius r	$4 \times \pi \times r^2$	$\frac{4}{3} \times \pi \times r^3$
Pyramid	base area B, height h		$\frac{1}{3} \times B \times h$

Vocabulary & Terms

The following list is representative of terminology used in the problems but **should not** be viewed as all-inclusive. It is recommended that coaches review this list with their Mathletes.

absolute difference
absolute value
acute angle
additive inverse *(opposite)*
adjacent angles
apex
arithmetic mean
arithmetic sequence
base ten
binary
bisect
box-and-whisker plot
center
chord
circumscribe
coefficient
collinear
common divisor
common factor
common fraction
complementary angles
congruent
convex
coordinate plane/system
coplanar
counting numbers
counting principle
diagonal of a polygon
diagonal of a polyhedron
digit sum
dilation
direct variation
divisor
domain of a function
edge
equiangular
equidistant
expected value
exponent
exterior angle of a polygon
factor
finite
frequency distribution
frustum
function
GCF (GCD)
geometric sequence
hemisphere
image(s) of a point(s) *(under a transformation)*
improper fraction
infinite series
inscribe
integer
interior angle of a polygon
intersection
inverse variation
irrational number
isosceles
lateral edge
lateral surface area
lattice point(s)
LCM
median of a set of data
median of a triangle
mixed number
mode(s) of a set of data
multiplicative inverse *(reciprocal)*
natural number
obtuse angle
ordered pair
origin
palindrome
parallel
Pascal's Triangle
percent increase/decrease
perpendicular
planar
polyhedron
polynomial
prime factorization
principal square root
proper divisor
proper factor
proper fraction
quadrant
quadrilateral
random
range of a data set
range of a function
rate
ratio
rational number
ray
real number
reciprocal *(multiplicative inverse)*
reflection
regular polygon
relatively prime
revolution
right angle
right polyhedron
rotation
scalene triangle
scientific notation
sector
segment of a circle
segment of a line
semicircle
semiperimeter
sequence
set
significant digits
similar figures
slope
space diagonal
square root
stem-and-leaf plot
supplementary angles
system of equations/inequalities
tangent figures
tangent line
term
transformation
translation
triangular numbers
trisect
twin primes
union
unit fraction
variable
whole number
y-intercept

SOLUTIONS

The solutions provided here are only *possible* solutions. It is very likely that you or your students will come up with additional—and perhaps more elegant—solutions. Happy solving!

Mixture Stretch

1. For the mixture to be 25% less sweet, it must have 25% less sugar; 25% is 1/4 of the full amount of sugar, which is 4 teaspoons. Therefore, Ming needs $1/4 \times 4 = \mathbf{1}$ teaspoon less sugar.

2. The fraction of the mixture that will consist of cherry candies and lime candies is $2/5 + 1/4 = (8 + 5)/20 = 13/20$. Grape candies make up the remaining $1 - 13/20 = 20/20 - 13/20 = \mathbf{7/20}$.

3. The concrete has $1 + 3 + 3 = 7$ parts. Therefore, the aggregate accounts for **3/7** of the mixture.

4. The total number of ounces of ingredients in the recipe is $12 + 4 + 7 + 4 + 5 = 32$ ounces. Since there are 16 ounces in a pound, this recipe makes $32/16 = 2$ pounds of stew. Since there are 7 ounces of potatoes in 2 pounds of stew, it follows that twice the amount of stew, or $2 \times 2 = 4$ pounds of stew, will contain $2 \times 7 = \mathbf{14}$ ounces of potatoes.

5. The 1-gallon mixture, which is 4% bleach, contains $0.04 \times 1 = 0.04$ gallons of bleach. The 2-gallon mixture, which is 10% bleach, contains $0.10 \times 2 = 0.20$ gallons of bleach. The $1 + 2 = 3$-gallon mixture contains $0.04 + 0.20 = 0.24$ gallons of bleach. The percent of the 3-gallon mixture that is bleach is $0.24/3 = 0.08 = \mathbf{8}\%$.

6. If we think of Myrna's mixture as having four parts, then it is 1/4 cashews, 1/4 almonds and 2/4 peanuts. One pound of Myrna's mixture contains 1/4 pound of cashews, which cost \$2.36 per pound, 1/4 pound of almonds, which cost \$1.48 per pound, and 1/2 pound of peanuts, which cost \$0.98 per pound. The cost to make a pound of Myrna's mixture is $1/4 \times 2.36 + 1/4 \times 1.48 + 1/2 \times 0.98 = 0.59 + 0.37 + 0.49 = \$1.45$. Myrna should charge 120% of that amount to make a 20% profit. That would be $1.2 \times 1.45 = \mathbf{\$1.74}$. Alternatively, 1 pound of cashews, 1 pound of almonds and 2 pounds of peanuts combined make a 4-pound mixture that has cashews, almonds and peanuts in the ratio 1:1:2. The cost of this 4-pound mixture is $2.36 + 1.48 + 2 \times 0.98 = \$5.80$, and $5.80/4 = \$1.45$ per pound. To include a 20% profit, the charge for a pound of Myrna's mixture should be $1.2 \times 1.45 = \mathbf{\$1.74}$.

7. We know that x gallons of the 80/20 mixture contain $0.8x$ gallons of detergent. In the 5-gallon bucket, this amount of detergent must account for 60% of the mixture, or $0.6 \times 5 = 3$ gallons. Therefore, $0.8x = 3$, so $x = 3.75$ gallons. The amount of pure water that Manny adds to each 5-gallon bucket, then, is $5 - 3.75 = \mathbf{1.25}$ gallons.

8. From the previous problem, we know that 3.75 gallons of the 80/20 mixture is added to each 5-gallon bucket. Therefore, the 15-gallon bucket will provide enough of the 80/20 mixture to make $15 \div 3.75 = \mathbf{4}$ buckets of the 60/40 mixture described in the previous problem.

9. The paint mixture has 6 parts, with white paint accounting for 1 part, or 1/6 of the mixture. If 1/6 of the 3-gallon mixture must be white paint, then Dara will need $1/6 \times 3 = 1/2$ gallon of white paint. Since there are 4 quarts in a gallon, 1/2 gallon of white paint is equivalent to $1/2 \times 4 = \mathbf{2}$ quarts.

10. The densities of the red, yellow and brown sand are 4 g/cm^3, 5 g/cm^3 and 6 g/cm^3, respectively. Therefore, 2 cm^3 of red sand has mass $2 \times 4 = 8$ g; 7 cm^3 of yellow sand has mass $7 \times 5 = 35$ g; and 5 cm^3 of brown sand has mass $5 \times 6 = 30$ g. Thus, the total mass of Arthur's sand sculpture is $8 + 35 + 30 = 73$ g. Since the total volume of sand in the sculpture is $2 + 7 + 5 = 14$ cm^3, the average density of the sculpture is $73/14 \approx \mathbf{5.2}$ g/cm^3.

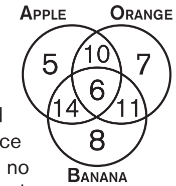
Statistics Stretch

11. If the mean score for the 28 students is 72, then the sum of the 28 scores must be $72 \times 28 = 2016$. Awarding an additional four points to each of 7 students increases that total by $7 \times 4 = 28$ points. The 28 scores now have a sum of $2016 + 28 = 2044$, and an average of $2044 \div 28 = \mathbf{73}$. Alternatively, we could recognize that the new total score will be $28 \times 72 + 7 \times 4 = 28 \times 72 + 28 = 28 \times 73$. Now, it becomes clear that dividing this product by 28 will result in an average score of $(28 \times 73) \div 28 = \mathbf{73}$.

12. Your first inclination might be to grab a calculator and add the numbers of pages to get $127 + 152 + 182 + 225 + 257 + 263 + 282 + 297 + 337 + 351 + 368 + 375 = \mathbf{3216}$ pages.

13. Adding the values that exceed 275 pages, we get $282 + 297 + 337 + 351 + 368 + 375 = 2010$ pages. That represents $2010/3216 = \mathbf{5/8}$ of the pages.

14. We can solve this problem by using a Venn diagram, with three circles representing the three fruits: apple, orange and banana. We are told that 6 of the 61 students who got fruit with lunch received an apple, an orange and a banana, so we place 6 at the intersection of all three circles. We are also told that 16 students got an apple and an orange. Since the intersection of the apple circle and the orange circle must reflect this, and since we already know that 6 of the students who got an apple and an orange also received a banana, 16 − 6 = 10 students must have gotten an apple and an orange but no banana. Similarly, since we are told that 17 students got an orange and a banana, 17 − 6 = 11 students must have gotten an orange and a banana but no apple. Likewise, since we are told that 20 students got an apple and a banana, 20 − 6 = 14 students must have gotten an apple and a banana but no orange. Finally, we place 5 in the apple circle and 7 in the orange circle to represent the numbers of students who got only an apple and only an orange. As the figure shows, we have accounted for 6 + 10 + 11 + 14 + 5 + 7 = 53 of the 61 students who got fruit. Therefore, the remaining 61 − 53 = 8 students got only a banana. We now see that 6 + 14 + 11 + 8 = 39 bananas were taken, 6 + 10 + 11 + 7 = 34 oranges were taken, and 6 + 10 + 14 + 5 = 35 apples were taken. Therefore, bananas represent 39/(39 + 34 + 35) = 39/108 ≈ 0.361 ≈ **36**% of the fruit taken by students.



15. If the 5 + 6 = 11 eighth graders account for half the club members, then the club must have a total of 11 × 2 = 22 members. If the seventh-grade beginners account for a third of the beginners, then the 1 + 5 = 6 sixth- and eighth-grade beginners must account for the other two-thirds of the beginners. Thus, the chess club has (3/2) × 6 = 9 beginners and 22 − 9 = 13 advanced chess players. So, the fraction of chess club members that are advanced chess players is **13/22**.

16. For the given set of numbers, A = (13 + 16 + 18 + 23 + 25 + 28 + 30 + 31)/8 = 184/8 = 23, M = (23 + 25)/2 = 48/2 = 24, and R = 31 − 13 = 18. Therefore, $A + M - R$ = 23 + 24 − 18 = **29**.

17. According to the graph, the 25 cat owners surveyed own a total of 6 × 1 + 9 × 2 + 6 × 3 + 3 × 4 + 1 × 5 = 6 + 18 + 18 + 12 + 5 = 59 cats. The mean number of cats per person is 59/25 = **2.36** cats.

18. The mean score for all 44 competitors is the sum of the scores divided by 44. Likewise, the mean score for the 16 highest scorers is the sum of their scores divided by 16, and the mean of the remaining 44 − 16 = 28 competitors is the sum of their scores divided by 28. Since we know the mean score for all the competitors is 28, the sum of all 44 scores is 28 × 44 = 1232. Also, we know that the 28 competitors who are not among the highest 16 scorers have a mean score of 20, which means the sum of their scores is 20 × 28 = 560. It follows, then, that the sum of the 16 highest scores is 1232 − 560 = 672, and the mean of these scores is 672/16 = **42**.

19. The total cost of the 6 bats Bryce ordered is 6 × 29.95 = $179.70. Since baseballs are sold in packs of 12 and Bryce ordered a total of 60 baseballs, he must have ordered 60 ÷ 12 = 5 packs. The total cost of the five packs of baseballs is 5 × 39.95 = $199.75. The total cost of the 8 gloves Bryce ordered is 8 × 69.95 = $559.60. The total cost of the equipment that Bryce ordered is 179.70 + 199.75 + 559.60 = $939.05. So, the portion of the total cost of Bryce's order attributed to the gloves is 559.60/939.05 ≈ 0.5959 ≈ **60**%.

20. From February to April the price increased by (75 − 60)/60 = 15/60 = 1/4. From April to June the price increased by (80 − 75)/75 = 5/75 = 1/15. The absolute difference is |1/4 − 1/15| = |15/60 − 4/60| = 11/60 ≈ 0.183 ≈ **18.3**%.

Pascal's Triangle Stretch

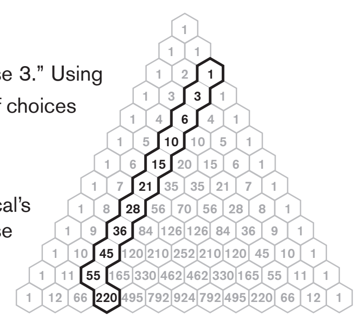
21. The greatest entry in row 15 of Pascal's triangle occurs twice, as $\binom{15}{7} = \binom{15}{8}$ = 15!/(7!8!) = **6435**. Alternatively, you could fill in the rest of the entries in the Pascal's triangle provided and see that the two center entries have the value **6435**.

22. The entries of row 14 of Pascal's triangle are 1, 14, 91, 364, 1001, 2002, 3003, 3432, 3003, 2002, 1001, 364, 91, 14, 1. These entries alternate odd, even, odd, even, odd, even, ... So, there are 8 odd entries and **7** even entries. (*Note:* There are many fascinating patterns involving the even and odd entries in Pascal's triangle. Try looking at the even and odd entries in other rows to see if you can spot any patterns!)

23. Since the sum of the entries in the nth row of Pascal's triangle is equal to 2^n, it follows that the sum of the entries in row 12 of Pascal's triangle is 2^{12} = **4096**.

24. We are looking for the number of ways to choose three objects from a collection of eight objects, or "8 choose 3." Using Pascal's triangle, we can determine the value of $\binom{8}{3}$ by locating entry 3 of row 8, which is 56. Therefore, the number of choices for three starters from eight players is **56** choices.

25. Recall that we can find the triangular numbers on the diagonal starting from either entry of 1 in row 2 of Pascal's triangle. The first 10 triangular numbers are 1, 3, 6, 10, 15, 21, 28, 36, 45 and 55. We can find the sum of these numbers by using the hockey stick identity. As the figure shows, the sum of the first 10 triangular numbers is 1 + 3 + 6 + 10 + 15 + 21 + 28 + 36 + 45 + 55 = **220**.



26. We are expanding a binomial of the form $(a + b)^8$. In this case, $a = x$ and $b = 2$, and we are looking for the coefficient of the a^3b^5 term. To determine the coefficient by using Pascal's triangle we look at entry 3 of row 8, which is 56. Therefore, the term will be $56a^3b^5$, and substituting x for a and 2 for b yields $56x^32^5$, which simplifies to $1792x^3$. So, the coefficient of the x^3 term in the expansion of $(x + 2)^8$ is **1792**.

27. We are expanding a binomial of the form $(a + b)^4$. In this case, $a = 2x$ and $b = y$. To determine the coefficients of the expansion, we look at row 4 of Pascal's triangle. The expanded form is $1a^4b^0 + 4a^3b^1 + 6a^2b^2 + 4a^1b^3 + 1a^0b^4$. When we substitute $2x$ for a, we see that the sum of the coefficients will be $1 \times 2^4 + 4 \times 2^3 + 6 \times 2^2 + 4 \times 2^1 + 1 \times 2^0 = 16 + 32 + 24 + 8 + 1 =$ **81**. Alternatively, we can obtain the sum of the coefficients by evaluating $(2x + y)^4$ when $x = y = 1$. We get $(2 + 1)^4 = 3^4 =$ **81**.

28. The 2023 terms of the expansion have the form $a_0x^{2022} + a_1x^{2021} + a_2x^{2020} + \cdots + a_{2021}x^1 + a_{2022}x^0$. The coefficients are given by the entries 0 through 2022 in row 2022 of Pascal's triangle. As we can see from the earlier rows, the coefficients are symmetric about the middle entry or median of each row and increase as we move inward from 1. Therefore, by symmetry, the greatest coefficient of a term in this sum of 2023 terms occurs for $a_{1011}x^{1011}$, whose value is entry 1011 of row 2022 of Pascal's triangle. Therefore, $b =$ **1011**.

Flips	Outcomes	Pascal's Triangle
1	H T	1 1
2	HH HT, TH TT	1 2 1
3	HHH HHT, HTH, THH HTT, THT, TTH TTT	1 3 3 1

29. As the table shows, the outcomes for the combinations of heads (H) and tails (T) when flipping a fair coin follow a pattern in Pascal's triangle. We are interested in the number of times a fair coin lands heads up at least as many times as it lands tails up when it is flipped four times. In other words, we need to determine the number of outcomes in which the coin lands heads up two or more times. If the pattern continues, we expect the numbers of outcomes of four flips that involve 4 heads up, 3 heads up, 2 heads up, 1 heads up and 0 heads up to be 1, 4, 6, 4 and 1, respectively. These are the entries of row 4 of Pascal's triangle, which have a sum of $2^4 = 16$. There are $1 + 4 + 6 = 11$ outcomes in which the coin lands heads up two or more times. Therefore, the probability is **11/16**.

30. It would be very slow to look for solutions to $\binom{n}{k} = 3003$ by checking each possible value of n, since clearly $\binom{3003}{1} = 3003$, meaning there is a solution in row 3003. Instead, we will look at possible values of k. Because $\binom{n}{k} = \binom{n}{n-k}$ we will focus on small values of k (namely, $k \le n/2$). For $k = 1$, we know $n = 3003$ works, so we get two solutions, $\binom{3003}{1} = \binom{3003}{3002} = 3003$. For $k = 2$, we need to solve $n(n - 1)/2 = 3003 \rightarrow n^2 - n = 6006 \rightarrow n^2 - n - 6006 = 0$. Using the quadratic formula to solve this equation, we get the positive solution $n = 78$. So, $\binom{78}{2} = \binom{78}{76} = 3003$. For $k = 3$, we need to solve $n(n - 1)(n - 2)/6 = 3003$. Start testing with $n = 27$, and we get $\binom{27}{3} = 2925$ and $\binom{28}{3} = 3276$. So, there is no integer solution for $k = 3$. For $k = 4$, we need to solve $n(n - 1)(n - 2)(n - 3)/24 = 3003$. Start testing with $n = 17$, and we get $\binom{17}{4} = 2380$ and $\binom{18}{4} = 3060$. Again, there is no integer solution. For $k = 5$, we need to solve $\binom{n}{5} = 3003$. Start testing with $n = 15$, and we get $\binom{15}{5} = 3003$. So, we have $\binom{15}{5} = \binom{15}{10} = 3003$. For $k = 6$, we need to solve $\binom{n}{6} = 3003$. Start testing with $n = 14$, and we get $\binom{14}{6} = 3003$. So, we have $\binom{14}{6} = \binom{14}{8} = 3003$. Since the two symmetric solutions we've just found are already so close together that there cannot be any additional values between them, it follows that there are no additional n and k values that work for $k \ge 7$. Thus, we have identified the **8** times that 3003 appears in Pascal's triangle.

Warm-Up 1

31. We can determine the value of 1,000,000 − 128,000 by using the standard subtraction algorithm. Doing so, we get **872,000**.

32. A cube with edges of length 3 units is composed of $3 \times 3 \times 3 =$ **27** unit cubes.

33. The smallest prime number is 2, the next-smallest is 3, and the third-smallest is 5. When we divide the sum of the first two primes by the third, we get $(2 + 3) \div 5 = 5 \div 5 = 1$ with a remainder of **0**.

34. Recall that the sum of the interior angle measures of a quadrilateral is always 360 degrees. A property of parallelograms is that opposite angles are congruent. So, $m\angle W = m\angle Y = 80$ degrees, and $m\angle X = m\angle Z = (360 - 160)/2 = 200/2 =$ **100** degrees. Alternatively, you may recall that in a parallelogram, consecutive angles are supplementary. Thus, $m\angle X = 180 - 80 =$ **100** degrees.

35. Using the order of operations, we evaluate the expression as follows: $[(6 \div 2) \times 3] + [(8 \div 4) \times 2] = (3 \times 3) + (2 \times 2) = 9 + 4 =$ **13**.

36. The nine single-digit positive integers are 1, 2, 3, 4, 5, 6, 7, 8 and 9. Of these, four are prime: 2, 3, 5, 7. Therefore, five are not prime numbers. The probability that the digit Yasuko randomly selects is not prime, then, is **5/9**.

37. Recall that multiplying by 1/10 is the same as dividing by 10, and a shortcut when dividing by a power of 10, say 10^x, is to move the decimal point to the left x places. Likewise, the shortcut when multiplying by a power of 10 is to move the decimal point to the right x places. So, we have $0.001 \div 10^1 \times 10^5 = 0.0001 \times 10^5 =$ **10**.

38. To determine the value of $3(4x + 5y) - 2(7x - 3y)$ when $x = -2$ and $y = 3$, we need to substitute −2 for each x and substitute 3 for each y in the expression. Doing so, we get $3(4 \times (-2) + 5 \times 3) - 2(7 \times (-2) - 3 \times 3)$. Now we evaluate the expression using the order of operations as follows: $3(-8 + 15) - 2(-14 - 9) = 3 \times 7 - 2 \times (-23) = 21 + 46 =$ **67**. Alternatively, we can distribute and simplify the original expression to get $12x + 15y - 14x + 6y = -2x + 21y$. Now, substituting −2 for x and 3 for y, we get $-2 \times (-2) + 21 \times 3 = 4 + 63 =$ **67**.

39. If we simplify 4/18, we get 2/9, and we can rewrite the proportion as 2/9 = a/27. Now we see that 27 = 9 × 3, so a = 2 × 3 = **6**. Alternatively, we could cross-multiply to get the equation 18a = 4(27). So, 18a = 108 and a = 108 ÷ 18 = **6**.

40. If 24 daifuku require 3 cups of sugar, then we can divide 24 by 3 to find the unit rate 8 daifuku per 1 cup of sugar. To find the number of cups of sugar needed for 64 daifuku, we divide 64 by 8, the number of daifuku per cup of sugar. The result is 64 ÷ 8 = **8** cups of sugar.

Warm-Up 2

41. Louisa will be away from the house for 0:14 + 3:27 + 0:14 = 3:55, or 5 minutes less than 4 hours. Since she left at 12:17 p.m., she will return home at 4:17 − 0:05 = **4:12** p.m.

42. Using the order of operations, we get 7 − (3 − 4) + 11 = 7 − (−1) + 11 = 7 + 1 + 11 = **19**.

43. The "least positive integer that is divisible by 4, 6 and 10" is another name for the least common multiple (LCM) of 4, 6 and 10. The prime factorizations of 4, 6 and 10 are 2^2, 2 × 3 and 2 × 5, respectively. To determine the LCM, we identify all the prime factors, which are 2, 3 and 5. Then we multiply the highest power of each prime factor. In this case, we get 2^2 × 3 × 5 = **60**.

44. The perimeter of a square of side length s is 4s. In this case s = 99 inches. So, the square has a perimeter of 99 × 4 = 396 inches. But we are asked to determine how many yards are in the perimeter of this square. Since there are 12 inches in a foot and 3 feet in a yard, it follows that there are 12 × 3 = 36 inches in a yard. Therefore, the perimeter of the square is 396 ÷ 36 = **11** yards.

45. The Schwartzes used 7/8 − 1/4 = 7/8 − 2/8 = 5/8 tank of gas on the trip. Since the gas tank holds 16 gallons, they used 5/8 × 16 = **10** gallons.

46. Although there is not just one sequence that starts with the given terms, we are asked to find a pattern and extend it. We see that this is not an arithmetic sequence because there is not a common difference between consecutive terms; nor is it a geometric sequence, because there is not a common ratio between consecutive terms. However, we notice that 3 is 2 more than 1, 7 is 4 more than 3, 13 is 6 more than 7, and 21 is 8 more than 13. To get the second through fifth terms of the sequence 2, 4, 6 and 8, respectively, were added to the previous term. If this arithmetic pattern of differences continues, then to go from the fifth to the eighth term, we need to add 10, then 12, then 14, or 10 + 12 + 14 = 36 in total. The eighth term of the sequence, then, is 21 + 36 = **57**. Alternatively, notice that subtracting 1 from each term yields the sequence 0, 2, 6, 12, 20, …, for which the nth term is just $n(n − 1)$. Therefore, the eighth term of the given sequence is 8 × 7 + 1 = 56 + 1 = **57**.

47. We can solve this equation by distributing, then combining like terms and, finally, solving for x. Doing so, we get $5(x + 2) − 3(x − 8) = 16 \rightarrow 5x + 10 − 3x + 24 = 16 \rightarrow 2x + 34 = 16 \rightarrow 2x = −18 \rightarrow x =$ **−9**.

48. Andrew and Ben mowed 1/2 + 1/3 = 3/6 + 2/6 = 5/6 of the lawn, and then Andrew mowed the remaining 1 − 5/6 = 1/6 of the lawn in 12 minutes. To mow the entire lawn by himself, at that rate, would have taken Andrew 12 × 6 = **72** minutes.

49. Recall that 125% = 1.25 = 5/4, and 25% = 1/4. We are told that (5/4)n = 30, so n = 30 × 4/5 = 24, and (1/4)n = 1/4 × 24 = **6**. Alternatively, since 25% is 1/5 of 125%, it follows that 25% of n is 1/5 of 125% of n, namely 1/5 × 30 = **6**.

50. Don can make 6 × 3 = 18 outfits from his 6 shirts and 3 pairs of trousers. But since he chooses not to wear the black shirt with the black trousers, that leaves Don **17** outfits.

Warm-Up 3

51. The number 1 appears one time in the numbers 1, 10 and 12, and two times in the number 11, so it is painted on the face of the clock **5** times.

52. The expression can be rewritten as 5 + 5 + 5 − 5 − 6 + 6 − 6 + 6. We now see that we are subtracting 6 twice and adding 6 twice, and these operations cancel each other out. Then we are left with 5 + 5 + 5 − 5 = **10**.

53. Point D has coordinates (6, 1) and their sum is 6 + 1 = **7**.

54. The expression simplifies as follows: $\left(1-\frac{1}{2}\right)^2\left(1-\frac{1}{3}\right)^2 = \left(\frac{1}{2}\right)^2\left(\frac{2}{3}\right)^2 = \frac{1}{4} \times \frac{4}{9} = \mathbf{\frac{1}{9}}$.

55. To find the sum of these numbers, we can add vertically, lining up the decimal points, to get **6.666**. Alternatively, you might notice that each addend has the digits 0, 1, 2 and 3, one in each of the ones, tenths, hundredths and thousandths places, and no digit appears in the same place-value position more than once. So, the sums of the digits in the ones, tenths, hundredths and thousandths places are all 6, resulting in a sum of **6.666**.

56. The primes less than 50 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43 and 47. Counting them, we find $p(50)$ = **15** primes.

57. Since $1\frac{1}{4}$ cups = $\frac{5}{4}$ cups, which is the amount of flour needed to make a casserole that serves 5 people, it follows that $\frac{1}{4}$ cup flour is needed per person. Therefore, to make enough casserole to serve 12 people, Maria will need to use $12 \times \frac{1}{4}$ = **3** cups of flour.

58. The prime factorization of 24 is $2^3 \times 3$ and the prime factorization of 40 is $2^3 \times 5$. The LCM is $2^3 \times 3 \times 5 = x$ and the GCF is $2^3 = y$. Therefore, $x/y = (2^3 \times 3 \times 5)/2^3$ = **15**.

59. The angle with degree measure x and the angle with measure $2x + 6$ are supplementary, so $x + 2x + 6 = 180$. Combining like terms and solving for x, we get $3x + 6 = 180$, so $3x = 174$ and $x = 174/3$ = **58** degrees.

60. We could check all of the two-digit prime numbers to see which of them are one greater than a perfect square. But instead let's check the two-digit perfect squares, since there are far fewer of them. The table shows each two-digit square n and $n + 1$. There are two primes here that are one greater than a perfect square: 17 and 37. The greatest of these is **37**.

n	$n+1$
16	**17**
25	26
36	**37**
49	50
64	65
81	82

Warm-Up 4

61. Since there are 9 calories per gram of fat, a cheeseburger that contains 63 calories of fat must have $63 \div 9 = 7$ grams of fat. Therefore, the total grams of fat in the burgers consumed by the 6 Mathletes is 7×6 = **42** grams.

62. The positive odd integers form a sequence with 1, 3, 5 and 7 as the first four terms. Notice that the 1st term is 1 less than 2(1); the 2nd term is 1 less than 4 = 2(2); the 3rd term is 1 less than 6 = 2(3); and the 4th term is 1 less than 8 = 2(4). From this, we can conclude that the nth term will have value $2n - 1$. Therefore, the 40th term in this sequence is $2(40) - 1 = 80 - 1$ = **79**.

63. We can use the distance formula to determine the distance d between C(2, 1) and D(5, 5). We have $d = \sqrt{(5-2)^2 + (5-1)^2} = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25}$ = **5** units. Alternatively, you might recognize that segment CD is the hypotenuse of a right triangle with legs of lengths 3 units and 4 units, making it a 3-4-5 right triangle. Thus, segment CD, the hypotenuse, has length **5** units.

64. If this pattern continues, we expect the value of $11{,}111^2$ to be 123,454,321. The sum of these digits is **25**.

65. Using the order of operations, we evaluate the expression as follows: $1 + (2 \times 3) - 4 + (5 \times 6) - 7 + (8 \times 9) = 1 + 6 - 4 + 30 - 7 + 72 = 109 - 11$ = **98** .

66. The increase of 28 – 20 = 8 people represents an increase in membership of 8/20 = 40/100 = **40**%.

67. Recall that the area of a rectangle is the product of its length and its width. So, the area of this rectangle is $2\frac{3}{4} \times 3\frac{2}{5} = \frac{11}{4} \times \frac{17}{5} = \frac{187}{20} = \mathbf{9\frac{7}{20}}$ in^2.

68. For this kind of puzzle, it helps to work out the pairs of numbers that produce each sum. The sum 3 can be made with 1 and 2, the sum 4 with 1 and 3, the sum 5 with 1 and 4 or 2 and 3, the sum 6 with 2 and 4, and the sum 7 with 3 and 4. Using this information, we can see that the sum of 7 in the top row requires 3 and 4, which means the sum of 4 in the third column will have to supply the 1 for the top row, and the sum of 3 in the last column will have to supply the 2 for the top row. The sum of 7 in the third row requires 3 and 4, and since there is already a 3 in the third column, the 4 must go in the third column and the 3 in the last column of the third row. The sum of 6 in the bottom row requires 2 and 4, and now the only option is to put the 2 in the third column and the 4 in the last column of the bottom row. The remainder of the grid can be filled in as shown, but we've already determined that the number that must be in the shaded cell is **4**.

3	4^{7}	1^{4}	2^{3}
4^{6}	2^{3}	3	1
2	1	4	3^{7}
1	3^{4}	2	$\mathbf{4}^{6}$

69. Since $1 + 2/3 = 3/3 + 2/3 = 5/3$, and $2 - 3/4 = 8/4 - 3/4 = 5/4$, the expression can be rewritten as $5/3 \div 5/4 = (5/3) \times (4/5)$ = **4/3**.

70. The symbol ☼ indicates a made-up operation with two inputs. Starting inside the parentheses, evaluate the expression as follows: 2 ☼ (2 ☼ 4) = 2 ☼ $\frac{(2+8)^2}{4}$ = 2 ☼ $\frac{10^2}{4}$ = 2 ☼ $\frac{100}{4}$ = 2 ☼ 25 = $\frac{(2+8)^2}{25} = \frac{100}{25}$ = **4**.

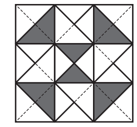
Warm-Up 5

71. The NSL team earned $29 \times 3 + 10 \times 2 + 4 \times 1 = 87 + 20 + 4$ = **111** points.

72. Subtracting these numbers vertically, lining up the decimal points and using the standard subtraction algorithm, we get 48.55 – 47.37 = 1.18. But we are asked to present the answer in the form of a common fraction, so we have 1.18 = 118/100 = **59/50**.

73. Since 1.21 = 121/100, its reciprocal is 100/121. So, $s = \sqrt{(100/121)} = \sqrt{100}/\sqrt{121} = \mathbf{10/11}$.

74. Since the mean of the 4 test scores is 95, the sum of those scores must be 4 × 95 = 380. Suppose three students scored 100. This yields the minimum possible score for the fourth student, which is 380 − 300 = **80** points.



75. As shown here, the figure can be divided into 36 congruent right triangles. Since 10 of these congruent triangles are shaded, the ratio of the shaded area to the whole is 10/36 = **5/18**.

76. According to the table, there are a total of 6 + 4 + 5 + 5 = 20 students on the playground. Since 5 of them are wearing shorts but no hat, the probability is 5/20 = **1/4**.

77. Evaluating these expressions and dividing, we get [−8 × (−4) − (−8)] ÷ [−6 + (−4)] = (32 + 8) ÷ (−6 − 4) = 40 ÷ (−10) = **−4**.

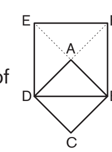
78. Let t represent the amount that Theo paid. Then the amount Simon paid is $(2/3)t$ and the combined amount they paid is $t + (2/3)t = (3/3)t + (2/3)t = (5/3)t$. Since we are told they paid a total of $15, we have the equation $(5/3)t = 15$. Solving for t, we get $t = 15 \times (3/5) = 9$. So, Theo paid $9 and Simon paid 15 − 9 = $6. Therefore, Theo paid 9 − 6 = $**3** or $**3.00** more than Simon. Alternatively, we know that for every $3 that Theo paid, Simon paid 2/3 that amount, $2. So, for every $5 portion of the bill, Theo paid $1 more than Simon. Since the bill was $15, which is three $5 portions, Theo paid $**3** or $**3.00** more than Simon.

79. The tick marks on six of the sides indicate that those segments all have the same length, which is 24 inches, or 24 ÷ 12 = 2 feet. The solid will be exactly half of a cube with side length of 2 feet and will have volume (1/2) × 2 × 2 × 2 = **4** ft^3.

80. Notice that 202,020,202,020 = 100,010,001 × 2020 and 202,120,212,021 = 100,010,001 × 2021. So, rewriting the expression, we get 202,020,202,020 × 2021 − 202,120,212,021 × 2020 = 100,010,001 × 2020 × 2021 − 100,010,001 × 2021 × 2020 = **0**.

Warm-Up 6

81. Simplifying, we see that $A = 2^3 - 3^2 + 4(5 + 1) = 8 - 9 + 4 \times 6 = 8 - 9 + 24 = 23$ and $B = 7^2 - 2(3 + 1)^2 = 49 - 2 \times 4^2 = 49 - 2 \times 16 = 49 - 32 = 17$. So, $A - B = 23 - 17 = \mathbf{6}$.

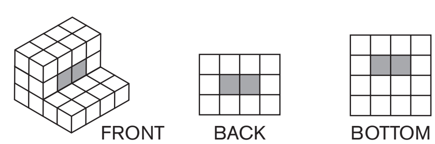


82. If we add the two dashed lines in the figure shown here, we divide the figure into 6 congruent right triangles. Square ABCD is made of 2 of these triangles whereas square BDEF is made of 4 of them. The ratio of their areas, then, is 2 to 4 or **1/2**.

83. The first ten prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29. In an ordered list of ten numbers, the median is the mean of the two middle terms. In this case, the median is (11 + 13) ÷ 2 = 24 ÷ 2 = **12**.

84. Notice that MATH is contained in SAMANTHA, and we know that M + A + T + H = 85. So, we must find the value of 85 + S + A + N + A. Since M = 8, we know that S = 3M − 15 = 3 × 8 − 15 = 24 − 15 = 9. Since H = 1, we know that A = 3H = 3 × 1 = 3 and N = 2H + S = 2 × 1 + 9 = 2 + 9 = 11. Therefore, SAMANTHA has the value 85 + 9 + 3 + 11 + 3 = **111**.

85. We could simply calculate $10^3 + 4^2 = 1000 + 16 = 1016$, and then divide to see that 1016 ÷ 8 = 127, with a remainder of **0**. Alternatively, we can find the remainder of $(10^3 + 4^2) \div 8$ this way. We know that $8 = 2^3$, $10^3 = (2 \times 5)^3 = 2^3 \times 5^3$ and $4^2 = (2^2)^2 = 2^4$, so we can rewrite the problem as $(2^3 \times 5^3 + 2^4) \div 2^3$. Notice that 2^3 is a divisor of both $2^3 \times 5^3$ and 2^4, so the remainder is **0**.



86. The figure shown here illustrates the location of the 6 unit cubes with exactly one painted face. The probability of randomly selecting one of these unit cubes is 6/32 = **3/16**.

87. The reciprocal of x is $1/x$. We know that $x = 2 \times 1/x$, so $x^2 = x(2/x) = 2$. Therefore, $x^4 = (x^2)^2 = 2^2 = \mathbf{4}$.

88. If the robotic vacuum cleaner can vacuum one room in 3/10 hour, it can vacuum 10/3 rooms in one hour, and 3 × 10/3 = **10** rooms in 3 hours.

89. For Nya, $N - 6 \times 13 = 25$, so $N = 25 + 78 = 103$. For Mya, $M + 8 \times 13 = 25$, so $M = 25 - 104 = -79$. Thus, $N + M = 103 + (-79) = \mathbf{24}$. Alternatively, the sum of Mya's and Nya's numbers increases by 8 − 6 = 2 at each step, for a total increase of 13 × 2 = 26. The sum of their numbers ends at 25 + 25 = 50, so the sum would have started at 50 − 26 = **24**.

90. If we find each of the sums in the parentheses, we have the product $\frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times \cdots \times \frac{19}{18} \times \frac{20}{19}$. Notice that nearly all of the numerators appear as a denominator of another fraction in the product. When matched, these pairs are equivalent to 1, so we can "cancel" them, leaving just 20/2 = **10**.

Warm-Up 7

91. If 7 students account for 25%, or 1/4, of Ms. Norton's students, then the total number of students in her class is 7 × 4 = **28** students.

92. The isosceles triangle is inscribed in a semicircle, so we know that the altitude of the triangle is the same as the radius, which is 10 cm. And since the base is the diameter of the semicircle, the area of the triangle is 1/2 × 20 × 10 = **100** cm^2.

93. Since $1.4 \times 10^5 = 14 \times 10^4$, we have $\frac{14\times10^4}{7\times10^2} = 2 \times 10^2 =$ **200**.

94. According to the figure, Maeve has 4 × 9 = 36 tiles, and she paints 1/3 × 36 = 12 tiles to create a design. Liam has 2 × 9 = 18 tiles, and he paints 5/9 × 18 = 10 tiles to create a design. The combined number of painted tiles in Maeve's and Liam's designs is 12 + 10 = **22** tiles.

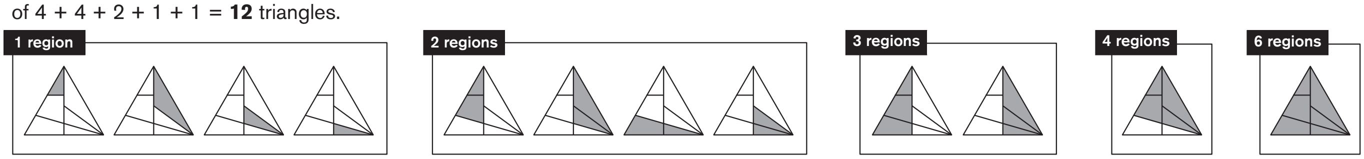
95. We are told that $a/b = 2/3$. Cross-multiplying, we see that $3a = 2b$ and $a = 2b/3$. We are also told that $a + b = 100$. Substituting for a, we have $2b/3 + b = 100$. Multiplying through by 3, we get $2b + 3b = 300$. So, $5b = 300$, and $b = 300 \div 5 =$ **60**. Alternatively, the sum of the numerator and denominator of 2/3 is 5, and we want to write an equivalent fraction in which the sum is 100. So, we can scale up by multiplying both the numerator and denominator by 100 ÷ 5 = 20. That gives us $a/b = (2 \times 20)/(3 \times 20) = 40/60$, so $b =$ **60**.

96. The unique mode of the five integers is 5, so the list must contain at least two 5s. The median is also 5, so the third integer in the ordered list must be 5. The mean is 5, so the sum of the five integers must be 5 × 5 = 25. We are looking for the greatest possible value in this list. To make the fifth integer as large as possible, we must make the first and second integers as small as possible. They must have values 1 and 2, respectively, in order for the mode of 5 to be unique, giving us 1, 2, 5, __, __. We must also make the fourth integer as small as possible, namely 5 (creating the desired mode), giving us 1, 2, 5, 5, __. Since the five integers must have a sum of 25, it follows that the fifth integer is 25 − (1 + 2 + 5 + 5) = 25 − 13 = 12, giving us 1, 2, 5, 5, 12. So, the greatest possible value of an integer in this list is **12**.

97. The weights in the bag have measures 1, 2, 4 and 5 ounces. The pairs of weights with a combined measure of 6 ounces, the weight of the toy car, are the 1- and 5-ounce weights and the 2- and 4-ounce weights. Since each pair can be removed in two different orders, there are a total of 2 × 2 = 4 ways to randomly remove two weights with a combined weight of 6 ounces. There are 4 × 3 = 12 ways of removing two weights, without replacement. Therefore, the desired probability is 4/12 = **1/3**.

98. The numerator $7^4 - 3^4$ is the difference of two squares, namely $(7^2)^2 - (3^2)^2$, and can be rewritten as $(7^2 + 3^2)(7^2 - 3^2)$. So, we have $\frac{\cancel{(7^2+3^2)}(7^2-3^2)}{\cancel{7^2+3^2}} = 7^2 - 3^2 = 49 - 9 =$ **40**.

99. As these figures show, there are 4 triangles made up of a single region of the original figure. There are 4 triangles made up of two regions, and there are 2 triangles made up of three regions. There is 1 triangle made up of four regions, and there is 1 triangle made up of six regions. That's a total of 4 + 4 + 2 + 1 + 1 = **12** triangles.



100. The playlist can start with any song, so there are 10 choices for the 1st song. But since the songs must alternate between ones Sara added and ones Ben added, there are only 5 choices for the 2nd song. Then there are 4 choices each for the 3rd and 4th songs, 3 choices each for the 5th and 6th songs, 2 choices each for the 7th and 8th songs and 1 choice each for the 9th and 10th songs. Thus, the total number of possible song arrangements for their playlist is 10 × 5 × 4 × 4 × 3 × 3 × 2 × 2 × 1 × 1 = **28,800** arrangements. Alternatively, since there are 5! = 120 arrangements of Sara's songs and 5! = 120 arrangements of Ben's songs, it follows that there are 120 × 120 = 14,400 arrangements of alternating songs if one of Sara's songs is played first and another 14,400 arrangements if one of Ben's songs is played first. That's a total of 14,400 + 14,400 = **28,800** arrangements.

Warm-Up 8

101. One-third of two-fifths of 90 is 1/3 × 2/5 × 90 = **12**.

102. We can divide the figure into two rectangular prisms, one with volume 4 × 3 × 1 = 12 $units^3$, and the other with volume 4 × 3 × 2 = 24 $units^3$. The total combined volume of the solid is 12 + 24 = **36** $units^3$.

103. From the information given, we can write the following equation: $n \div (-6) + 6 = 3$. So, $n \div (-6) = -3$, and $n = -3 \times (-6)$, which means $n =$ **18**.

104. To evaluate this continued fraction, we start at the bottom: 1 + 1/3 = 3/3 + 1/3 = 4/3. Since 1/(4/3) = 3/4, we next add 1 + 3/4 = 4/4 + 3/4 = 7/4. Again, since 1/(7/4) = 4/7, we next add 1 + 4/7 = 7/7 + 4/7 = **11/7**.

105. From the table, we see that the slope is $(7 - 2)/(4 - 1) = 5/3$. Rewriting $2x - y = 4$ in slope-intercept form, we get $y = 2x - 4$. So, the slope is 2. The absolute difference of these slopes is $|2 - 5/3| = 6/3 - 5/3 =$ **1/3**.

106. The value of $2^5 \times 4^{-2}$ is $32 \times 1/4^2 = 32/16 =$ **2**. Alternatively, since $4^{-2} = (2^2)^{-2} = 2^{-4}$, we have $2^5 \times 2^{-4} =$ **2**.

107. Since 365 is one more than a multiple of 7, the day of the week of Armin's birthday will shift one day forward each regular year and two days forward during a leap year (except in 2020, since his birthday occurs after February 29, 2020). The table shows the day of Armin's birthday in the 6 years it takes for his birthday to again be on a Saturday. Armin's birthday will next be on a Saturday when he is 13 + 6 = **19** years old.

2021	2022	2023	2024	2025	**2026**
SUN	MON	TUE	THU	FRI	**SAT**

108. In a geometric sequence there is a common ratio r between consecutive terms. In this case, we know that $-1r = 3$, so $r = -3$. We know that $a = -9r$, so substituting for r, we get $a = -9 \times (-3) =$ **27**.

109. We don't need to calculate $10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$. We only need to determine the number of times 3 occurs as a factor. We have 3, $6 = 2 \times 3$ and $9 = 3^2$, so 3^4 is the greatest power of 3 that is a factor of 10! Thus, the greatest possible value of N is **4**.

110. ACD is a 30-60-90 right triangle with shorter leg CD of length 6 units. Therefore, by properties of special right triangles, hypotenuse AC has length $6 \times 2 = 12$ units. But AC is also the hypotenuse of $\triangle$ACE, which is a 45-45-90 right triangle. Again, by properties of special right triangles, it follows that $\text{CE} \times \sqrt{2} = 12$, so $\text{CE} = 12/\sqrt{2} \times \sqrt{2}/\sqrt{2} = 12\sqrt{2}/2 = 6\sqrt{2}$ units. Finally, CE is also the longer leg of 30-60-90 right triangle CEB. So, by properties of special right triangles, we have $\text{EB} \times \sqrt{3} = 6\sqrt{2}$, so $\text{EB} = 6\sqrt{2}/\sqrt{3} \times \sqrt{3}/\sqrt{3} = 6\sqrt{6}/3 =$ $\mathbf{2\sqrt{6}}$ units.

Warm-Up 9

111. There is a 5/12 probability that the first bow Jo Jo pulls out is red. There is a 4/11 probability that the second bow she pulls out is also red. That's a $5/12 \times 4/11 = 20/132 = \underline{10/66}$ probability that Jo Jo pulls out two red bows. There is a $4/12 = 1/3$ probability that Jo Jo first pulls out a white bow. There is a 3/11 probability that she next pulls out another white bow. That's a $1/3 \times 3/11 = \underline{6/66}$ probability that Jo Jo pulls out two white bows. There is a $3/12 = 1/4$ probability that the first bow Jo Jo pulls out is blue. There is a 2/11 probability that the second bow she pulls out is also blue. That's a $1/4 \times 2/11 = \underline{3/66}$ probability that Jo Jo pulls out two blue bows. Therefore, the probability that the two bows Jo Jo pulls out are the same color is $10/66 + 6/66 + 3/66 =$ **19/66**.

112. Distributing and solving for x, we get $(5/2)x - 2 = 3 \rightarrow (5/2)x = 5 \rightarrow x = 5 \times 2/5 =$ **2**.

113. There are $2^5 = 32$ outcomes for 5 flips of a fair coin. The only 2 outcomes without consecutive heads or consecutive tails are HTHTH and THTHT. Therefore, the desired probability is $2/32 =$ **1/16**. Alternatively, in order for no two consecutive coin flips to result in both heads or both tails, the 2nd flip must be different from the 1st flip ($P = 1/2$), the 3rd flip must be different from the 2nd flip ($P = 1/2$), the 4th flip must be different from the 3rd flip ($P = 1/2$) and the 5th flip must be different from the 4th flip ($P = 1/2$). Thus, the probability of this occuring is $1/2 \times 1/2 \times 1/2 \times 1/2 =$ **1/16**.

114. The figure is a right triangle with leg lengths x units and $3x$ units and hypotenuse of length 10 units. Using the Pythagorean theorem, we have $x^2 + (3x)^2 = 10^2$. Simplifying, we get $x^2 + 9x^2 = 100$, so $10x^2 = 100$. Now solving for x yields $x^2 = 10$, and $x =$ $\mathbf{\sqrt{10}}$.

115. Since $4 = 2^2$, we can rewrite 4^{12} as $(2^2)^{12} = 2^{24}$. Similarly, since $125 = 5^3$, we can rewrite 125^7 as $(5^3)^7 = 5^{21}$. So, we can rewrite and calculate as follows: $4^{12} \times 125^7 = 2^{24} \times 5^{21} = 2^3 \times 2^{21} \times 5^{21} = 2^3 \times (2 \times 5)^{21} =$ $\mathbf{8 \times 10^{21}}$.

116. Evan's rectangle has perimeter $46 \times 2 = 92$ cm, and it has area $90 \div 2 = 45$ cm^2. So, the product of the dimensions of Evan's rectangle is 45, and since the perimeter of a rectangle is twice the sum of the length and width, the sum of its dimensions must be 46. There is only one pair of positive integers that meet these criteria, 45 and 1. Thus, the smaller dimension of Evan's rectangle is **1** cm.

117. Let n and q represent the numbers of nickels and quarters, respectively, that Tonya found. We know that $0.05n + 0.25q = 2.25$, or $n + 5q = 45$. We also know that $n = 3q + 5$. Substituting for n in the first equation, we get $3q + 5 + 5q = 45 \rightarrow 8q + 5 = 45 \rightarrow 8q = 40 \rightarrow q = 5$. So, $n = 3q + 5 = 3 \times 5 + 5 = 20$. Tonya found 5 quarters and 20 nickels, for a total of 5 + 20 = **25** coins.

118. The slope of the line containing the points $(-7, 1)$ and $(5, 7)$ is $(7 - 1)/[5 - (-7)] = 6/12 = 1/2$. Since the point $(0, b)$ is also on this line, it follows that $(b - 7)/(0 - 5) = 1/2$. Cross-multiplying and solving for b, we get $2(b - 7) = -5 \rightarrow 2b - 14 = -5 \rightarrow 2b = 9 \rightarrow b =$ **9/2**.

119. Since $1841 \div 6 = 306$ R 5, it follows that $1841 = 6 \times 306 + 5 = 6n + k$. Therefore, $n = 306$ and $k = 5$. So, $n + k = 306 + 5 =$ **311**.

120. If the lamp's shade has a circumference that is three times that of the lamp's base, that means the diameter of the lamp's shade is also three times that of the lamp's base. For the lamp's shade, we are told that $C = \pi d = 18\pi$ inches, so $d = 18$ inches. It follows, then, that the lamp's base has diameter $18 \div 3 = 6$ inches and radius $6 \div 2 = 3$ inches. We are also told that the lamp's base has height 9 inches. So, the cylindrical brass lamp base has volume $V = \pi r^2 h = \pi \times 3^2 \times 9 = \pi \times 9 \times 9 =$ $\mathbf{81\pi}$ in^3.

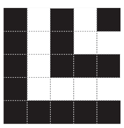
Warm-Up 10

121. Segments AC and EF divide square ABCD into three regions—isosceles right triangles ABC and DEF and isosceles trapezoid ACEF. The trapezoid's area is the area of square ABCD minus the combined areas of triangles ABC and DEF. Square ABCD has side length 8 cm and area $8 \times 8 = 64$ cm^2. Since AB = BC = 8 cm, it follows that $\triangle$ABC has area $1/2 \times 8 \times 8 = 32$ cm^2. Similarly, since E and F are the midpoints of sides CD and AD, respectively, it follows that DE = DF = $8 \div 2 = 4$ cm, and $\triangle$DEF has area $1/2 \times 4 \times 4 = 8$ cm^2. Therefore, trapezoid ACEF has area $64 - (32 + 8) = 64 - 40 =$ **24** cm^2.

122. Substituting -2 for x in the function $f(x) = 2x^3 - 5x^2 + 9x + 4$, we see that $f(-2) = 2 \times (-2)^3 - 5 \times (-2)^2 + 9 \times (-2) + 4 = 2 \times (-8) - 5 \times 4 + 9 \times (-2) + 4 = -16 - 20 + (-18) + 4 =$ **−50**.

123. There are ${}_6C_3 = 6!/(3! \times 3!) = (6 \times 5 \times 4)/(3 \times 2 \times 1) = 20$ subsets of three numbers chosen from the set {2, 3, 5, 7, 11, 13}. The least possible sum of a subset of three numbers from the set is $2 + 3 + 5 = 10$, and the greatest possible sum is $7 + 11 + 13 = 31$. So, we are looking for subsets of three numbers that have a sum of 12, 15, 18, 21, 24, 27 or 30. Shown here are the 7 subsets of three numbers that have a sum that is a multiple of 3. Therefore, the desired probability is **7/20**.

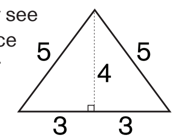
12	15	18	21	27
2 + 3 + 7	3 + 5 + 7	2 + 3 + 13	3 + 5 + 13	3 + 11 + 13
		2 + 5 + 11	3 + 7 + 11	



124. As shown, the square can be divided into 25 congruent squares, 15 of which are shaded. The fraction shaded is 15/25 = **3/5**.

125. Raising a quantity to the 1/2 power is equivalent to taking the square root of that quantity. So, $\left[\left(\frac{2}{3}\right)^2 \div 4\right]^{\frac{1}{2}} = \sqrt{\left(\frac{2}{3}\right)^2 \div 4} = \sqrt{\frac{4}{9} \times \frac{1}{4}} = \sqrt{\frac{1}{9}} = \mathbf{\frac{1}{3}}$.

126. For each triangular base, we can draw an altitude to the side of length 6 cm, as shown, to create two 3-4-5 right triangles. We now see that each triangular base has height 4 cm and area $1/2 \times 6 \times 4 = 12$ cm^2. So, the bases contribute $2 \times 12 = 24$ cm^2 to the total surface area of the prism. The area of the three rectangular faces of the prism can be calculated by multiplying the perimeter of the triangular base, which is $5 + 5 + 6 = 16$ cm, by the height of the prism, which is 3 cm, to get $16 \times 3 = 48$ cm^2. Thus, the total surface area of the prism is $24 + 48 =$ **72** cm^2.

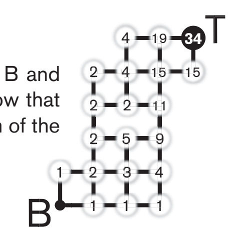


127. For a number to be evenly divisible by 9, the sum of its digits must be a multiple of 9. Since we are looking for the smallest five-digit number and since $4 + 6 = 10$, it follows that the five digits must have a sum of 18, and the other three digits must add to 8. The smallest positive five-digit number is 10,000, so let's make two of the digits 1 and 0, which means the final digit must be $8 - (1 + 0) = 7$. Using the digits 0, 1, 4, 6 and 7 we get the smallest possible number with the given characteristics, **10,467**.

128. Because section A has a central angle of measure 90 degrees, we know that its area is 90/360 = <u>1/4</u> of the spinner. The combined areas of sections A and B account for $1/4 \times 2 = 2/4 = 1/2$ of the spinner. That means the combined areas of sections C, D and E also account for 1/2 of the spinner, with each section being $1/3 \times 1/2 =$ <u>1/6</u> of the spinner. Therefore, the probability that the spinner lands on section A or section D is $1/4 + 1/6 = 3/12 + 2/12 =$ **5/12**.

129. The geometric mean of two numbers A and B is the square root of their product, or $\sqrt{(AB)}$. We are told that the geometric mean of A and B is 4, or $\sqrt{(AB)} = 4$, so $AB = 16$. We are also told that the arithmetic mean of A and B is 5, so $(A + B) \div 2 = 5 \rightarrow A + B = 10 \rightarrow A = 10 - B$. Substituting, we get $(10 - B) \times B = 16 \rightarrow -B^2 + 10B = 16 \rightarrow B^2 - 10B + 16 = 0$. We can solve for B by factoring the quadratic to get $(B - 8)(B - 2) = 0$. That means $B - 8 = 0$, so $B = 8$. Or $B - 2 = 0$, so $B = 2$. The larger of these two values is **8**.

130. In the figure, the numbers at each intersection indicate the number of ways to get to that intersection when starting from B and moving up and right to T. We determine the number of ways to get to each intersection by adding the values to the left and below that intersection since those are the segments of the path leading to that intersection. For example, the 9 at one intersection is the sum of the 5 to its left and the 4 below it. As the figure shows, there are **34** of the paths described.



Warm-Up 11

131. Given $f(x) = x^2$ and $g(x) = 2x - 1$, we find that $f(5) = 5^2 = 25$ and $g(8) = 2 \times 8 - 1 = 16 - 1 = 15$. Thus, $f(5) - g(8) = 25 - 15 =$ **10**.

132. The largest integer less than 200 is 199, so we'll start by checking to see if it is a nugget number. We know that we'll need a 9 as part of our sum because of the 9 in the units place. So, let's focus on writing $199 - 9 = 190$ as the sum of 6s, 9s and 20s. Notice that we can get 160 by adding eight 20s. So, now we need to determine if there is a way to write $190 - 160 = 30$ as the sum of 6s, 9s and 20s. Well, $9 + 6 = 15$ and $15 \times 2 = 30$. Therefore, the largest nugget number less than 200 is $20 + 20 + 20 + 20 + 20 + 20 + 20 + 20 + 9 + 9 + 9 + 6 + 6 =$ **199**.

133. First, note that $81^{\frac{3}{4}}$ can be rewritten as $(81^{\frac{1}{4}})^3$. Since raising a quantity to the 1/4 power is equivalent to taking the fourth root of the quantity, we have $(\sqrt[4]{81})^3 = [\sqrt{(\sqrt{81})}]^3 = (\sqrt{9})^3 = 3^3 =$ **27**.

134. There are no repeated letters in the word SQUARE, which means there are $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 =$ **720** arrangements of these six letters.

135. The sum of all seven numbers is 28, so we are looking for ways to create two groups, with the numbers of each group adding to $28 \div 2 = 14$. The group containing the 7 must have the remaining numbers sum to $14 - 7 = 7$. The only ways to do this are 6 + 1, 5 + 2, 4 + 3 and 4 + 2 + 1. Therefore, the number of ways to separate the given numbers into two groups so that the sum of the numbers in each group is the same is **4** ways.

136. Some students might just notice that $x = -1$ is a solution to the given equation and yields the result **2** when substituted into the given expression. Alternatively, squaring both sides of the equation gives us $\left(x + \frac{1}{x}\right)^2 = (-2)^2 \rightarrow x^2 + 2 + \frac{1}{x^2} = 4 \rightarrow x^2 + \frac{1}{x^2} = 2$. Then, squaring both sides of this equation gives us $\left(x^2 + \frac{1}{x^2}\right)^2 = 2^2 \rightarrow x^4 + 2 + \frac{1}{x^4} = 4 \rightarrow x^4 + \frac{1}{x^4} = \mathbf{2}$.

137. The bill is \$0.03 less than a whole number of dollars. This suggests that three containers of fries were ordered for \$0.99 each. They would account for \$2.97 of the cost, leaving 17.97 – 2.97 = \$15.00 spent on burgers. At \$2.50 per burger, there would be $15 \div 2.5 =$ **6** burgers.

138. Recall the formula for the surface area of a right circular cone $SA = \pi \times r^2 + \pi \times r \times \sqrt{(r^2 + h^2)}$. For this cone, $r = 4$ cm and $SA = 56\pi$ cm^2. Substituting the known values into the formula gives $56\pi = \pi \times 4^2 + \pi \times 4 \times \sqrt{(4^2 + h^2)}$. Simplifying and combining like terms yields $56 = 16 + 4 \times \sqrt{(16 + h^2)} \rightarrow 40 = 4 \times \sqrt{(16 + h^2)} \rightarrow 10 = \sqrt{(16 + h^2)}$. Squaring both sides, we get $100 = 16 + h^2$, so $h^2 = 84$, and $h = \sqrt{84} = \mathbf{2\sqrt{21}}$ cm.

139. We are looking for the number that is 3 more than the least common multiple (LCM) of 4, 5 and 6. Since $4 = 2^2$, 5 is prime and $6 = 2 \times 3$, it follows that the LCM of 4, 5 and 6 is $2^2 \times 3 \times 5 = 60$. So, the least positive integer that leaves a remainder of 3 when divided by each of 4, 5 and 6 is $60 + 3 =$ **63**.

140. The shortest altitude for a triangle is the one drawn perpendicular to the longest side. In this case, it would be the altitude drawn to the side of length 30 units. Since we have the lengths of all three sides of the triangle, we can use Heron's formula to determine its area. Then, once we know the area, we can substitute for the area A and base length b in the formula $A = 1/2 \times b \times h$ to find the height h. Recall that Heron's formula is $A = \sqrt{s(s-a)(s-b)(s-c)}$, where s is the semiperimeter and a, b and c are the side lengths. For the given triangle, we have $s = (18 + 24 + 30) \div 2 = 72 \div 2 = 36$ units. So, we have $A = \sqrt{36(36-18)(36-24)(36-30)} = \sqrt{36 \times 18 \times 12 \times 6} = \sqrt{6 \times 6 \times 9 \times 2 \times 2 \times 6 \times 6} = 36 \times 3 \times 2 = 216$ units^2. Now, substituting into the other area formula, we get $216 = 1/2 \times 30 \times h$. So, $15h = 216$ and $h = 216/15 =$ **72/5** units. Alternatively, if we recognize that the triangle with side lengths 18, 24 and 30 units is a 3-4-5 right triangle scaled up by a factor of 6, we can find the length of the shortest altitude a different way. The area of a 3-4-5 right triangle is $1/2 \times 3 \times 4 = 6$ units^2. The shortest altitude is the one drawn perpendicular to the hypotenuse. If its length is h, then $1/2 \times 5 \times h = 6$, so $5 \times h = 12$, and $h = 12/5$. Scaling this up makes the shortest altitude of the given triangle $6 \times 12/5 =$ **72/5** units.

Workout 1

141. We know that $6 = 2 \times 3$ and $9 = 3^2$, so the LCM of 6 and 9 is $2 \times 3^2 = 18$. That means that the numbers that have 6 and 9 as divisors are precisely the multiples of 18. The sum of the multiples of 18 less than 100 is $18 + 36 + 54 + 72 + 90 =$ **270**.

142. We know the output, so we can work in reverse to determine the value of the input. We start with $12G = 144$, so $G = 144 \div 12 = 12$. Next, we have $(1/10)F = 12$, so $F = 12 \times 10 = 120$. Next, $E + 100 = 120$, so $E = 120 - 100 = 20$. Then, $(5/6)D = 20$, so $D = 20 \times 6/5 = 24$. Next, $6C = 24$, so $C = 24 \div 6 = 4$. Next, $B^2 = 4$, so $B = \sqrt{4} = \pm 2$. Since we are told that A is a positive integer, it must be that $(1/5)A = 2$, and $A = 2 \times 5 =$ **10**.

143. The arithmetic mean is $(3 + 66 + 999) \div 3 = 1068 \div 3 =$ **356**.

144. The distance from mile marker 7 to mile marker 29 is 29 – 7 = 22 miles. At 72 mi/h, it will take Grace $22 \div 72 \approx$ **0.31** hours.

145. The piano has a total of 52 + 36 = 88 keys, and the white keys account for $52/88 \approx 0.59 =$ **59**% of the piano keys.

146. Since Rob weighs 80 kg and Kristen weighs 60 kg, it follows that Rob has $80 \times 39 - 60 \times 40 =$ **720** mL more blood plasma than Kristen.

147. At the rate of 5 times per second, to play a song that lasts 3 minutes 43 seconds, a CD will spin $(3 \times 60 + 43) \times 5 =$ **1115** times.

148. There are several pairs of positive integers whose sum is 34, and the closer the integers are in value, the greater their product will be. This is demonstrated in the graph of $y = x(34 - x) = -x^2 + 34$, which is a parabola that opens downward, with a vertex at $x = 17$, midway between its x-intercepts at $x = 0$ and $x = 34$. Therefore, the greatest product is $17 \times 17 =$ **289**.

149. Each × above a number represents a student in Ms. Coleman's homeroom who reported reading that number of books. So, the three ×s above 5 on the number line indicate that three students reported reading 5 books. The total number of books the 20 students reported reading was $1 \times 1 + 2 \times 2 + 4 \times 3 + 4 \times 4 + 3 \times 5 + 2 \times 6 + 2 \times 7 + 2 \times 8 = 90$ books. That's an average of $90 \div 20 =$ **4.5** books per student.

150. Two angles are complementary if the sum of their degree measures is 90 degrees. Angles A and B are complementary, so $5x - 6 + 3x = 90$. Simplifying and solving for x, we get $8x - 6 = 90 \rightarrow 8x = 96 \rightarrow x = 12$. The measure of angle A, then, is $5x - 6 = 5 \times 12 - 6 =$ **54** degrees.

Workout 2

151. We know that 1/12 foot = 1 inch = 2.54 cm. So, 50 cm is equivalent to $50 \times (1/12)/2.54 \approx$ **1.64** feet.

152. Some students will quickly come up with the integer 22, since $2 + 2 = 2 \times 2$, but is this the only two-digit integer with the stated property? Let A represent the tens digit and B the ones digit. We are looking for a two-digit number for which $A + B = AB \rightarrow A = AB - B \rightarrow A = B(A - 1) \rightarrow B = A/(A - 1)$. That means A must be divisible by one less than itself, which is true only when $A = 2$ and $B = 2 \div (2 - 1) = 2$. Thus, the number of two-digit positive integers with the stated property is **1** integer.

153. Alistair has at least one penny, one nickel, one dime and one quarter, so let's set aside these 4 coins that are worth 1 + 5 + 10 + 25 = 41 cents. The greatest possible value of coins will occur when the other 36 coins are all quarters. Since 4 quarters are equivalent to \$1.00, 36 quarters equals $36 \div 4$ = \$9.00. The greatest possible value of the coins in Alistair's pocket, then, is **\$9.41**.

154. We are told that Sanjay runs 80% = 0.8 = 4/5 as far as Jerome. Conversely, Jerome runs 5/4 = 1.25 = **125**% as far as Sanjay.

155. The first five positive cubes are 1, 8, 27, 64 and 125. The product of all five of these cubes is a common multiple, but not the least common multiple, since 8 is a factor of 64. Since 27, 64 and 125 are relatively prime, we get an LCM of $27 \times 64 \times 125$ = **216,000**.

156. This triangle has as its vertices the origin and the x- and y-intercepts for the line $3x + 2y = 12$. To find the x-intercept, we let $y = 0$, and get $3x = 12$, so $x = 4$. To find the y-intercept, we let $x = 0$, and get $2y = 12$, so $y = 6$. Therefore, the triangle has vertices (0, 0), (4, 0) and (0, 6). It follows, then, that this triangle has base length 4 units, height 6 units and area $1/2 \times 4 \times 6$ = **12** units^2.

157. Gene worked 5 hours 15 minutes, or 5.25 hours. Doug worked 4 hours 45 minutes, or 4.75 hours. Pat worked 4 hours 15 minutes, or 4.25 hours. The combined number of hours they worked is 5.25 + 4.75 + 4.25 = **14.25** hours.

158. Because $1000 \div 28$ = 35 R 20, we conclude that there are 35 multiples of 28 between 1 and 1000. The median of these will be the 18th value, which is 28×18 = **504**.

159. The hexagon of area 160 units^2 is, essentially, a 12.5-unit by15.8-unit rectangle, of area $12.5 \times 15.8 = 197.5$ units^2, with a 5-unit by a-unit corner rectangle, of area $5a$ units^2, removed. The corner rectangle must have area 197.5 − 160 = 37.5 units^2. So, we have $5a = 37.5$, and $a = 37.5 \div 5$ = **7.5** units.

160. The first 10 primes are 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29. Here are the sums of the digits of these primes, written in the same order: 2, 3, 5, 7, 2, 4, 8, 10, 5 and 11. We have circled the sums of the digits for the **7** primes that are optimus primes.

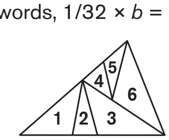
Workout 3

161. The easiest way to compare the given values is to put them all in the same form, so we'll convert them all to decimals since we're asked to express our answer in decimal form. We have $7/3 = 2.\overline{3}$ and 1 1/4 = 1.25. Therefore, the four values in increasing order are 1.25, 1.5, 2 and $2.\overline{3}$. There are an even number of numbers, so the median is the mean of the two middle values, 1.5 and 2, which is $(1.5 + 2) \div 2$ = **1.75**.

162. We are initially told that Marco is older than 20 but younger than 60. Reading on, we see that Marco must be at least 32 years old in order for the square of his age to be a four-digit number. But the sum of the digits of his age is 8, resulting in three possibilities for Marco's age: 35, 44 and 53. Since we are told that the digits of his age are different and his age is not prime, that rules out 44 and 53 as possible ages. So, we conclude that Marco is **35** years old.

163. Notice that the segment from the end of the shadow to the top of the flagpole is the hypotenuse of a 30-60-90 right triangle. By properties of special right triangles, we know that this segment has length twice that of the shadow, or 16 feet. Now, with leg length 8 feet and hypotenuse length 16 feet, the Pythagorean theorem gives the height of the flagpole as $\sqrt{(16^2 - 8^2)} \approx$ **13.9** feet.

164. In the 10 hours from 9:00 a.m. to 7:00 p.m., the number of blueberries in the bowl is reduced by half $10 \div 2 = 5$ times. That means the number of blueberries in the bowl at 7:00 p.m. is $1/2^5 = 1/32$ of the number of blueberries b that Sam originally placed in the bowl. In other words, $1/32 \times b = 5$, so $b = 5 \times 32 = 160$. So, Sam filled the bowl at 9:00 a.m. with **160** blueberries.



165. Recall that adjacent angles that form a line are supplementary and have measures that sum to 180 degrees. Recall, also that the sum of the measures of the angles of a triangle is 180 degrees. Using these properties and the given angle measures, we can determine that the missing angle measures for each triangle in the figure, must be 36, 72 or 108 degrees, and see that the 6 non-overlapping triangles in the figure are all isosceles. In the figure shown here, we have numbered these triangles 1 through 6, to more easily identify the remaining isosceles triangles. There are 3 isosceles triangles composed of two smaller triangles: 1-2, 2-3 and 4-5. Then there are 2 isosceles triangles composed of three smaller triangles: 1-2-3 and 4-5-6. We can't forget that the largest triangle, which contains the 6 smaller triangles, is isosceles. That's a total of 6 + 3 + 2 + 1 = **12** isosceles triangles.

166. There are 10,101 multiples of 99 that are less than 1,000,000, since 10101 × 99 = 999,999. There are 20,202 multiples of 99 that are less than 2,000,000, since 20,202 × 99 = 1,999,998. That means the number of integers between 1,000,000 and 2,000,000 that are divisible by 99 must be 20,202 − 10,101 = **10,101** integers.

167. The volume of an acre-foot is 66 × 660 × 1 = 43,560 ft^3. Recall that 12 inches = 1 foot, so 1728 in^3 = 1 ft^3. Now since 1 gallon = 231 in^3 we see that 1 acre-foot of water contains 43,560 × 1728 ÷ 231 = 325,851.4… ≈ **326,000** gallons.

168. The 33rd positive integer multiple of 3 is 99. So, 33 of the positive integers from 1 to 100, inclusive, are multiples of 3. That's 33/100 = **33**%.

169. After 30 students move from Team A to Team B, twice as many students are on Team A as on Team B. So, Team A has 2/3 × 300 = 200 students, and Team B has 1/3 × 300 = 100 students. That means the number of students originally on Team B was 100 − 30 = **70** students.

170. Since the mean number of cats for all 50 apartments is 0.44 cats, there must be 50 × 0.44 = 22 cats in the apartment building. Only 50 − 32 = 18 apartments actually have cats, so the mean number of cats in those apartments is 22 ÷ 18 = **1.2** cats.

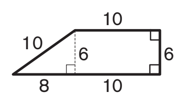
Workout 4

171. Cross-multiplying, we get $5t = 4(3t + 1) \rightarrow 5t = 12t + 4 \rightarrow -7t = 4 \rightarrow t =$ **−4/7**.

172. The five algebra books can be arranged in 5! = 120 ways, and the four geometry books can be arranged in 4! = 24 ways. Since there are two ways to arrange the two sets of books on the shelf, it follows that Mr. Scott can arrange all nine books on the shelf in 120 × 24 × 2 = **5760** ways.

173. The seven equations can be rewritten and added, as shown, to get $A + B + C + D + E + F + G =$ **1**.

$$\begin{array}{rcr}
A + B - C & = & 1 \\
B + C - D & = & -1 \\
C + D - E & = & 1 \\
D + E - F & = & -1 \\
E + F - G & = & 1 \\
-A + F + G & = & -1 \\
A - B + G & = & 1 \\
\hline
A + B + C + D + E + F + G & = & \mathbf{1}
\end{array}$$



174. If we drop a perpendicular from the left end of the short base to the long base below, we cut the trapezoid into a rectangle and a right triangle. The triangle must be a 6-8-10 right triangle, satisfying the Pythagorean theorem with $6^2 + 8^2 = 10^2$. The area of the rectangle is 6 × 10 = 60 cm^2, and the area of the triangle is 1/2 × 6 × 8 = 24 cm^2. The total area of the trapezoid, then, is 60 + 24 = **84** cm^2.

175. Let n represent the number of tests that Danielle took, and let s represent the sum of all the test scores. We are told that the mean of Danielle's test scores is 85, so we have $s \div n = 85$, or $s = 85n$. We are also told that her lowest test score is 61 and that the mean of the scores except the lowest score is 88. Thus, we have $(s - 61) \div (n - 1) = 88 \rightarrow s - 61 = 88(n - 1) \rightarrow s - 61= 88n - 88 \rightarrow s = 88n - 27$. We can now set the two expressions for s equal to one another to get $85n = 88n - 27$. Simplifying and solving for n, we get $3n = 27$, so $n = 9$. So, Danielle took **9** tests.

176. The figure shows 8 hexagons, which have 8 × 6 = 48 vertices, and 6 squares, which have 6 × 4 = 24 vertices. That's a total of 48 + 24 = 72 vertices on the two-dimensional net. When the net is folded into the three-dimensional truncated octahedron, two hexagons and one square meet at each vertex. Thus, the number of vertices on the solid is 72 ÷ 3 = **24** vertices. Alternatively, since each vertex of the solid is a vertex of exactly one square, it is sufficient to count just the vertices of the 6 squares. Doing so yields a total of 4 × 6 = **24** vertices for the solid.

177. The ratio of milk to flour in the recipe is 1.5 cups to 2 cups. Let a represent the amount of flour needed to make a batch of pudding using 7.75 cups of milk. We can set up the proportion $1.5/2 = 7.75/a$. Cross-multiplying and solving for a yields $1.5a = 2 \times 7.75$, so $a = 2 \times 7.75 \div 1.5 = 10.\overline{3}$. So, Jamie will need to use **10.3** cups of flour.

178. When Shawna turned 21 years old, Shelby was 21 ÷ 3 = 7 years old. In y years, Shawna will be twice as old as Shelby. We can write the following equation: $21 + y = 2(7 + y)$. Simplifying and solving for y, we get $21 + y = 14 + 2y \rightarrow y = 7$. So, in 7 years, Shelby will be 7 + 7 = 14 years old and Shawna will be 21 + 7 = **28** years old, which is twice Shelby's age.

179. The reciprocal of a number x is $1/x$. We are told that $x + 1/x = -17/4$. Let's first get rid of the fractions by multiplying both sides of this equation by $4x$. We get $4x(x + 1/x) = 4x(-17/4) \rightarrow 4x^2 + 4 = -17x \rightarrow 4x^2 + 17x + 4 = 0$. Factoring the quadratic expression yields $(x + 4)(4x + 1) = 0$. So, $x + 4 = 0$ and $x = -4$, or $4x + 1 = 0$ and $x = -1/4$. The sum of these solutions is $-4 + (-1/4) = -16/4 - 1/4 =$ **−17/4**. Alternatively, we can use the fact that for any quadratic equation $ax^2 + bx + c = 0$, the sum of its solutions is $-b/a$. For the quadratic equation $4x^2 + 17x + 4 = 0$, $a = 4$ and $b = 17$, so the sum of its roots is $-b/a =$ **−17/4**.

180. At 11:45 a.m., the ratio of adults to children was 9 to 11, and there were a total of 9 + 11 = 20 people. At 12:00 p.m., there were 60 people, which is 3 times the number of people who were there at 11:45 a.m. Since the ratio of adults to children was again 9 to 11, there must have been 9 × 3 = 27 adults and 11 × 3 = 33 children there at 12:00 p.m. Between 11:45 a.m. and 11:50 a.m., 8 more children came to the movie, making the total number of children at 11:50 a.m. equal to 11 + 8 = 19 children. So, the number of children that came to the movie between 11:50 a.m. and 12:00 p.m. was 33 − 19 = **14** children.

Workout 5

181. Since a and d are only used once in the expression $ab + bc + cd$, they should have the smallest values, 2 and 3. This will also make the product bc as great as possible, namely 7 × 8 = 56. Now we have two options for $ab + cd$, either 2 × 7 + 8 × 3 = 38, or 2 × 8 + 7 × 3 = 37. Using the first option, we get the greatest possible value of $ab + bc + cd$, which is 2 × 7 + 7 × 8 + 8 × 3 = **94**.

182. Recall that the formula for the volume of a sphere of radius r equals $V = 4/3 \times \pi \times r^3$. We are told that the ball has volume 137,250 cm^3. Substituting, we have $137{,}250 = 4/3 \times \pi \times r^3$. Solving for r yields $r^3 = 137{,}250 \times 3/4 \div \pi \rightarrow r = \sqrt[3]{(137{,}250 \times 3/4 \div \pi)}$. But we are asked to find the diameter of the ball, which is $d = 2r = 2 \times \sqrt[3]{(137{,}250 \times 3/4 \div \pi)} \approx$ **64** cm.

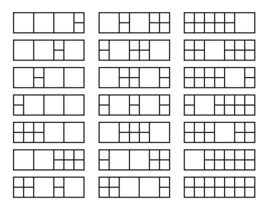
183. We need a row or column with two or more values to determine its missing values. That leaves only the fourth column of the square array. Since the numbers of the column must form an arithmetic progression, the missing term between 8 and 36 must be the mean of 8 and 36, or (8 + 36) ÷ 2 = 22. That means the common difference between values in this column is 22 − 8 = 14. Therefore, the values of the fourth column must be 8, 22, 36, 50, 64. Now we see that the common difference in the second row is 22 − 36 = −14, so the values must be 64, 50, 36, 22, 8. Next, moving to the fourth row, we see that the missing term between 66 and 50 must be (66 + 50) ÷ 2 = 58. That means the common difference between values in this row is 58 − 66 = −8, so the values must be 74, 66, 58, 50, 42. Repeating this method for the remaining columns results in the filled array shown, in which the greatest value is **79**.

59	42	25	8	−9
64	50	36	22	8
69	58	47	36	25
74	66	58	50	42
79	74	69	64	59

184. The circle has radius 6 units and area $\pi \times 6^2 = 36\pi$ units^2. The right triangle has an area of 1/2 × 6 × 6 = 18 units^2. The legs of the right triangle create a quarter-circle sector that has area $36\pi \div 4 = 9\pi$ units^2. The area of the shaded segment of the circle is the difference of the area of the quarter-circle sector and the area of the right triangle, or $9\pi - 18 \approx$ **10.3** units^2.

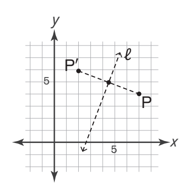
185. Let w represent the number of votes the winner received. Then the loser received $0.6w$, and $w + 0.6w = 432$. Simplifying and solving for w, we get $1.6w = 432$, so $w = 270$. That means the winner received 270 votes and the loser received 432 − 270 = 162 votes. That's a difference of 270 − 162 = **108** votes.

186. Michelle will earn the minimum amount when there are only four Tuesdays and only four Thursdays during the 31-day month. That will happen, for example, if the month starts on a Friday and ends on a Sunday. When that happens, Michelle will earn 4 × 112.50 + 4 × 135 = $**990** or $**990.00**.



187. The figure shows all **21** ways that the grid can be completely covered with 1 × 1 tiles and 2 × 2 tiles. Alternatively, let a_n be the number of ways to cover a rectangle that has dimensions 2 × n. Then a configuration of length n either ends with a 2 × 2 tile preceded by a valid configuration of length $n - 2$, or ends with a vertically stacked pair of 1 × 1 tiles preceded by a valid configuration of length $n - 1$. Therefore, $a_n = a_{n-2} + a_{n-1}$. Furthermore, clearly $a_1 = 1$ and $a_2 = 2$. We can then calculate the successive values recursively and find $a_3 = 3$, $a_4 = 5$, $a_5 = 8$, $a_6 = 13$ and $a_7 = 21$. Note that this is just the Fibonacci sequence. Thus, there are **21** ways to completely cover the grid with 1 × 1 tiles and 2 × 2 tiles.

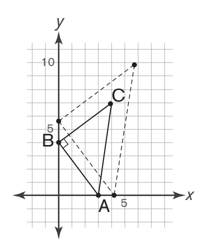
188. The first three terms of the sequence are $S_1 = 4$, $S_2 = 4 + (3 + 2)^2 = 29$ and $S_3 = 29 + (3 + 3)^3 =$ **245**.



189. The line ℓ must be perpendicular to the segment with endpoints P and P′, as shown. The slope of that segment is (6 − 4)/(2 − 7) = −2/5. The slope of line ℓ, then, will be the opposite reciprocal of the slope of that segment, namely **5/2**.

190. The ratio of vitamin oil to water in the mixture is 1 drop to 75 drops. One portion of the mixture, then, is 1 + 75 = 76 drops. A 1520-drop mixture has 1520 ÷ 76 = 20 portions. Since each portion contains 1 drop of vitamin oil, this mixture will contain **20** drops of vitamin oil.

Workout 6



191. A dilation of $\sqrt{2}$ about the origin will increase each of the triangle's side lengths by a factor of $\sqrt{2}$. Its area will then be $(\sqrt{2})^2 =$ 2 times that of the original triangle. The graph of △ABC shows that it is an isosceles right triangle with leg length 5 units and hypotenuse $5\sqrt{2}$ units. The area of △ABC, then, is 1/2 × 5 × 5 = 12.5 units^2. Therefore, the dilated triangle has area 12.5 × 2 = **25** units^2.

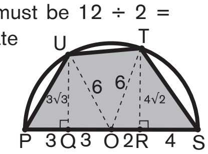
192. We need to award at least one prize for each value, so let's first set aside those 7 prizes, which total 1 + 3 + 15 + 60 + 120 + 360 + 1800 = \$2359. The remaining prize money is 10,000 – 2359 = \$7641. Note that \$7641 is divisible by 3, and all the prize values except \$1 are also divisible by 3, so if we award another \$1 prize, then we will have to award at least three \$1 prizes to restore divisibility by 3. But there is no point in using three \$1 prizes, because one \$3 prize has the same effect and uses fewer prizes. So, discard the \$1 prize. Now note that all the prizes that remain are divisible by 5 except the \$3 prize, but \$7641 is not divisible by 5. Neither is 7641 − 3 = \$7638, but 7641 − 2 × 3 = \$7635 is divisible by 5. So, we need at least two more \$3 prizes. If we use more than two more \$3 prizes we would need to use at least seven more \$3 prizes to get divisibility by 5, and there is no point in doing that because we could use one \$15 prize in place of five \$3 prizes. So, we are left with minimizing the number of prizes of values \$15, \$60, \$120, \$360 and \$1800 to achieve a total of \$7635. To make things easier, let's divide everything by 15 to simplify the arithmetic. That gives us prize values of \$1, \$4, \$8, \$24 and \$120 to add up to \$509. Following the same kind of reasoning, looking at divisibility by 4, we now must include exactly one more \$1 prize (which really represents a \$15 prize) in order to get a total of \$508 using \$4, \$8, \$24 and \$120 prizes. Again simplify arithmetic by dividing everything by 4, leaving prizes of values \$1, \$2, \$6 and \$30 to add up to \$127. The same reasoning tells us that we must include one more \$1 prize (which really represents a \$60 prize). Divide through by 2, leaving prizes of values \$1, \$3 and \$15 to add up to \$63. Looking at divisibility by 3 now forces us not to use the \$1 prize (which is really \$120), so we divide through by 3 to get prizes of values \$1 and \$5 to reach \$21. At this point, it is clear that we will use the \$1 prize (which is really \$360) and four \$5 prizes (which are really \$1800 prizes). In summary, we've awarded <u>1</u> prize worth \$1, <u>3</u> prizes worth \$3 each, <u>2</u> prizes worth \$15 each, <u>2</u> prizes worth \$60 each, <u>1</u> prize worth \$120, <u>2</u> prizes worth \$360 each and <u>5</u> prizes worth \$1800 each. That's a total of 1 + 3 + 2 + 2 + 1 + 2 + 5 = **16** prizes.

193. According to the rule, the estimated weight of this salmon is $28.5 \times 10.25^2 \div 775 \approx$ **3.9** pounds.

194. Right triangle PQR has vertices at P(a, 4), Q(a, −4) and R($-a$, −4). The triangle has a base of length $2a$ and a height of 8. Since its area is 12 units^2, we can write the equation $1/2 \times 2a \times 8 = 12$, so $8a = 12$, and $a = 3/2 =$ **1.5**.

195. We are looking for the LCM of these amounts of time: 50, 60, 90, 120 and 360 minutes. Since $50 = 2 \times 5^2$, $60 = 2^2 \times 3 \times 5$, $90 = 2 \times 3^2 \times 5$, $120 = 2^3 \times 3 \times 5$ and $360 = 2^3 \times 3^2 \times 5$, it follows that the LCM of these numbers is $2^3 \times 3^2 \times 5^2 = 1800$. That means that all five trains will next pass through the station at the same time after 1800 minutes = **30** hours.

196. Side PS of quadrilateral PSTU is a diameter of the semicircle and has a length of 3 + 5 + 4 = 12 units, so the radius must be 12 ÷ 2 = 6 units. That means QO = 6 − 3 = 3 units, and OR = 6 − 4 = 2 units. If we draw the two radii OU and OT, as shown, we create triangles POU and SOT. Triangle POU is equilateral, so by properties of 30-60-90 right triangles, we know that $UQ = 3\sqrt{3}$ units. We can use the Pythagorean theorem to determine TR. We have $2^2 + (TR)^2 = 6^2 \rightarrow 4 + (TR)^2 = 36 \rightarrow (TR)^2 = 32 \rightarrow TR = \sqrt{32} = 4\sqrt{2}$ units. The area of quadrilateral PSTU is the combined areas of △PQU, trapezoid QUTR and △SRT. Combined, the areas equal $1/2 \times 3 \times 3\sqrt{3} + 1/2 \times (3\sqrt{3} + 4\sqrt{2}) \times 5 + 1/2 \times 4 \times 4\sqrt{2} \approx$ **46.2** units^2.



197. If B is the midpoint of segment AD and AB = BD = 6, then AD = 6 + 6 =12. Next, we know that AD + DE = AE and AD = 3/4 × AE, so AE = 12 × 4/3 = 16 and DE = 16 − 12 = 4. That means BE = BD + DE = 6 + 4 = 10. Since BC + CE = BE, if BC = 1/5 × BE, then CE = 4/5 × BE. Thus, CE = 4/5 × 10 = **8** units.

198. If each bag contains 3/8 pound of cashews, Vick can make 33 ÷ 3/8 = 33 × 8/3 = 88 bags of cashews. He sells them for \$0.79 each, so that's 0.79 × 88 = \$69.52 in sales. Since he spent \$50 for the cashews, his profit is \$69.52 – \$50 = \$19.52. This profit represents 19.52/69.52 ≈ 0.281 = **28.1**% of the price Vick charges.

199. Let r and w represent the costs of a red bead and a white bead, respectively. We can write the following system of equations: $12r + 10w = 14$ and $8r + 15w = 13.50$. Dividing both sides of the first equation by 2 yields $6r + 5w = 7$. We can multiply this equation by 3 to get $18r + 15w = 21$. Subtracting the equations $18r + 15w = 21$ and $8r + 15w = 13.50$ yields $10r = 7.50$, so $r = 0.75$. Thus, the cost of a red bead is \$**0.75**.

200. The given equation can be rewritten as $x^2 + 2xy + y^2 = 8$. The trinomial on the left-hand side is the square of a binomial. We can rewrite the equation as $(x + y)^2 = 8$, so $x + y = \pm\sqrt{8} = \pm 2\sqrt{2}$. Thus, $|x + y| =$ $\mathbf{2\sqrt{2}}$.

PROBLEM INDEX

It is very difficult to categorize many of the problems in this handbook. MATHCOUNTS problems often straddle multiple categories and cover several concepts, but in this index, we have placed each problem in exactly one category and mapped it to exactly one Common Core State Standard (CCSS). **In this index, code 9 (3) 7.SP.3 would refer to problem #9 with difficulty rating 3 mapped to CCSS 7.SP.3.** The difficulty rating and CCSS mapping are explained below.

DIFFICULTY RATING: Our scale is 1-7, with 7 being most difficult. These general ratings are only approximations:

- **1, 2 or 3**: Appropriate for students just starting the middle school curriculum; 1 concept; 1- or 2-step solution.
- **4 or 5**: Knowledge of some middle school topics necessary; 1-2 concepts; multi-step solution.
- **6 or 7**: Knowledge of advanced middle school topics and/or problem-solving strategies necessary; multiple and/or advanced concepts; multi-step solution.

COMMON CORE: We align our problems to the NCTM Standards for Grades 6-8, however we also have mapped these problems to CCSS because 42 states, D.C., 4 territories and the Dept. of Defense Education Activity (DoDEA) have voluntarily adopted it. Our CCSS codes contain (in this order):

1. **Grade level** in the K-8 Standards for Mathematical Content (SMC). Courses that are in the high school SMC instead have the first letter of the course name.
2. **Domain** within the grade level or course and then the **individual standard**.

Here are 2 examples:

- *6.RP.3 → Standard #3 in the Ratios and Proportional Relationships domain of grade 6*
- *G-SRT.6 → Standard #6 in the Similarity, Right Triangles and Trigonometry domain of Geometry*

Some math concepts are not specifically mentioned in CCSS. For problems using these concepts, we use the code of a related standard, when possible. Some of our problems are based on concepts outside the scope of CCSS or are based on concepts in the K-5 SMC but are more difficult than a grade K-5 problem. When appropriate, we coded these problems SMP for the CCSS Standards for Mathematical Practice.

Sd

STATISTICS

11	(3)	6.SP.5
12	(2)	6.SP.4
16	(3)	6.SP.2
17	(3)	6.SP.5
18	(4)	7.SP.3
74	(3)	7.SP.3
83	(2)	6.SP.2
96	(3)	6.SP.2
143	(2)	6.SP.2
149	(2)	6.SP.4
158	(2)	6.SP.5
161	(2)	7.NS.2
170	(3)	6.SP.5
175	(3)	7.SP.3

Pg

PLANE GEOMETRY

34	(2)	8.G.5
59	(3)	8.G.5
75	(3)	SMP
82	(3)	SMP
92	(3)	7.G.6
110	(5)	G-SRT.6
114	(3)	8.G.7
140	(5)	7.G.6
150	(3)	7.G.5
159	(3)	7.G.6
165	(4)	SMP
184	(5)	G-C.2
196	(6)	G-SRT.6

Me

MEASUREMENT

44	(2)	5.MD.1
67	(3)	7.G.6
79	(4)	7.G.6
116	(4)	SMP
121	(2)	7.G.6
163	(4)	G-SRT.6
174	(3)	7.G.6
182	(3)	G-GMD.3
193	(2)	6.EE.7

Pr

PROPORTIONAL REASONING

40	(2)	6.RP.2
57	(3)	7.RP.1
88	(2)	6.RP.3
146	(2)	6.RP.1
147	(1)	7.RP.1
151	(3)	6.RP.3
167	(4)	6.RP.3
177	(3)	6.RP.3
180	(4)	SMP
190	(2)	6.RP.3

Cg

COORDINATE GEOMETRY

53	(1)	5.G.4
63	(1)	8.G.8
118	(3)	8.F.4
156	(4)	7.G.6
189	(4)	G-GPE.5
191	(5)	8.G.3
194	(4)	7.G.6

Sg

SOLID GEOMETRY

32	(1)	7.G.3
102	(2)	7.G.6
120	(4)	8.G.9
126	(5)	7.G.6
138	(4)	8.G.9
176	(5)	7.G.6

Pf — PERCENTS & FRACTIONS

1	(1)	6.RP.3
2	(2)	6.RP.3
3	(1)	6.RP.3
4	(3)	6.RP.3
5	(4)	6.RP.3
6	(4)	6.RP.3
7	(5)	6.RP.3
8	(4)	6.RP.3
9	(3)	6.RP.3
10	(4)	7.RP.1
13	(3)	6.RP.1
15	(4)	7.NS.3
19	(3)	6.RP.3
20	(5)	7.RP.3
23	(2)	SMP
45	(1)	6.NS.1
49	(2)	7.RP.3
66	(2)	7.RP.3
69	(3)	6.NS.1
78	(3)	7.RP.3
91	(1)	6.RP.3
94	(1)	6.NS.1
124	(3)	SMP
145	(2)	6.RP.3
154	(3)	6.RP.3
185	(4)	6.RP.3
197	(3)	7.NS.2
198	(4)	7.NS.3

Nt — NUMBER THEORY

33	(1)	6.NS.2
43	(2)	6.NS.4
54	(2)	6.EE.1
56	(2)	SMP
58	(3)	6.NS.4
60	(4)	SMP
77	(1)	7.NS.2
80	(4)	SMP
90	(3)	7.NS.3
93	(2)	8.EE.4
98	(4)	A-SEE.2
104	(3)	8.NS.1
106	(2)	6.EE.1
109	(4)	SMP
115	(4)	N-RN.2
125	(3)	N-RN.2
132	(3)	SMP
133	(5)	N-RN.2
139	(4)	SMP
141	(2)	6.NS.4
148	(2)	7.NS.1
152	(3)	6.EE.9
155	(3)	6.NS.4
160	(3)	6.NS.4
166	(3)	6.NS.4
168	(1)	6.NS.4
192	(4)	SMP
195	(4)	6.RP.3

Al — ALGEBRAIC EXPRESSIONS & EQUATIONS

26	(4)	A-APR.5
27	(4)	A-APR.5
28	(5)	A-APR.5
38	(2)	6.EE.2
39	(2)	6.EE.6
47	(2)	6.EE.6
70	(3)	6.EE.9
73	(2)	7.NS.2
81	(2)	6.EE.1
84	(3)	7.EE.4
87	(3)	6.EE.2
95	(3)	8.EE.8
103	(1)	6.EE.2
105	(3)	8.F.2
112	(2)	8.EE.7
119	(2)	SMP
122	(3)	8.F.1
129	(4)	A-REI.4
131	(3)	8.F.1
136	(4)	8.EE.2
142	(1)	SMP
164	(2)	7.NS.3
171	(3)	6.EE.6
173	(4)	8.EE.8
178	(3)	7.EE.4
179	(4)	A-REI.4
199	(4)	7.EE.4
200	(5)	A-REI.2

Pc — PROBABILITY, COUNTING & COMBINATORICS

24	(3)	S-CP.9
29	(4)	7.SP.8
36	(1)	7.SP.7
50	(3)	S-CP.9
76	(2)	7.SP.6
97	(4)	S-CP.6
100	(4)	S-CP.9
111	(4)	S-CP.6
113	(4)	7.SP.8
123	(3)	7.SP.8
128	(4)	S-CP.7
130	(4)	SMP
134	(3)	S-CP.9
172	(3)	S-CP.9
187	(5)	S-CP.9

Ps — PROBLEM SOLVING (MISCELLANEOUS)

21	(3)	SMP
22	(2)	SMP
25	(3)	SMP
30	(7)	S-CP.9
41	(1)	SMP
48	(3)	6.NS.1
51	(1)	SMP
86	(3)	SMP
89	(3)	7.EE.4
107	(3)	SMP
117	(4)	6.EE.7
127	(3)	6.NS.4
135	(3)	SMP
137	(3)	SMP
144	(2)	7.RP.1
153	(2)	SMP
157	(2)	7.NS.3
169	(4)	7.NS.3

Gm — GENERAL MATH

31	(1)	6.NS.3
35	(1)	3.OA.2
37	(1)	5.NBT.2
42	(2)	7.NS.1
52	(1)	7.NS.1
55	(1)	6.NS.3
61	(1)	7.RP.1
65	(2)	3.OA.8
71	(2)	SMP
72	(1)	6.NS.3
85	(2)	8.EE.1
101	(2)	6.NS.1

Lo — LOGIC

14	(5)	SMP
64	(2)	F-BF.2
68	(3)	SMP
99	(4)	SMP
162	(1)	SMP
181	(4)	SMP
186	(4)	SMP

Sp — SEQUENCES, SERIES & PATTERNS

46	(2)	F-IF.3
62	(1)	F-IF.3
108	(3)	F-BF.2
183	(3)	F-BF.2
188	(3)	F-IF.3

COACHES:
FIND PROBLEMS, ANSWERS, SOLUTIONS + PROBLEM INDEX FOR PROBLEMS 201–250 at www.mathcounts.org/coaches!

ANSWERS

MIXTURE STRETCH
1. 1* (1)
2. 7/20 (2)
3. 3/7 (1)
4. 14 (3)
5. 8 (4)
6. 1.74 (4)
7. 1.25 (5)
8. 4 (4)
9. 2 (3)
10. 5.2 (4)

STATISTICS STRETCH
11. 73 (3)
12. 3216 (2)
13. 5/8 (3)
14. 36 (5)
15. 13/22 (4)
16. 29 (3)
17. 2.36 (3)
18. 42 (4)
19. 60 (3)
20. 18.3 (5)

PASCAL'S TRIANGLE STRETCH
21. 6435 (3)
22. 7 (2)
23. 4096 (2)
24. 56 (3)
25. 220 (3)
26. 1792 (4)
27. 81 (4)
28. 1011 (5)
29. 11/16 (4)
30. 8 (7)

WARM-UP 1
31. 872,000 (1)
32. 27 (1)
33. 0 (1)
34. 100 (2)
35. 13 (1)
36. 5/9 (1)
37. 10 (1)
38. 67 (2)
39. 6 (2)
40. 8 (2)

WARM-UP 2
41. 4:12 (1)
42. 19 (2)
43. 60 (2)
44. 11 (2)
45. 10 (1)
46. 57 (2)
47. −9 (2)
48. 72 (3)
49. 6 (2)
50. 17 (3)

WARM-UP 3
51. 5 (1)
52. 10 (1)
53. 7 (1)
54. 1/9 (2)
55. 6.666 (1)
56. 15 (2)
57. 3 (3)
58. 15 (3)
59. 58 (3)
60. 37 (4)

WARM-UP 4
61. 42 (1)
62. 79 (1)
63. 5 (1)
64. 25 (2)
65. 98 (2)
66. 40 (2)
67. $9\frac{7}{20}$ (3)
68. 4 (3)
69. 4/3 (3)
70. 4 (3)

WARM-UP 5
71. 111 (2)
72. 59/50 (1)
73. 10/11 (2)
74. 80 (3)
75. 5/18 (3)
76. 1/4 (2)
77. −4 (1)
78. 3 or 3.00 (3)
79. 4 (4)
80. 0 (4)

WARM-UP 6
81. 6 (2)
82. 1/2 (3)
83. 12 (2)
84. 111 (3)
85. 0 (2)
86. 3/16 (3)
87. 4 (3)
88. 10 (2)
89. 24 (3)
90. 10 (3)

WARM-UP 7
91. 28 (1)
92. 100 (3)
93. 200 (2)
94. 22 (1)
95. 60 (3)
96. 12 (3)
97. 1/3 (4)
98. 40 (4)
99. 12 (4)
100. 28,800 (4)

WARM-UP 8
101. 12 (2)
102. 36 (2)
103. 18 (1)
104. 11/7 (3)
105. 1/3 (3)
106. 2 (2)
107. 19 (3)
108. 27 (3)
109. 4 (4)
110. $2\sqrt{6}$ (5)

WARM-UP 9
111. 19/66 (4)
112. 2 (2)
113. 1/16 (4)
114. $\sqrt{10}$ (3)
115. 8×10^{21} (4)
116. 1 (4)
117. 25 (4)
118. 9/2 (3)
119. 311 (2)
120. 81π (4)

WARM-UP 10
121. 24 (2)
122. −50 (3)
123. 7/20 (3)
124. 3/5 (3)
125. 1/3 (3)
126. 72 (5)
127. 10,467 (3)
128. 5/12 (4)
129. 8 (4)
130. 34 (4)

WARM-UP 11
131. 10 (3)
132. 199 (3)
133. 27 (5)
134. 720 (3)
135. 4 (3)
136. 2 (4)
137. 6 (3)
138. $2\sqrt{21}$ (4)
139. 63 (4)
140. 72/5 (5)

WORKOUT 1
141. 270 (2)
142. 10 (1)
143. 356 (2)
144. 0.31 (2)
145. 59 (2)
146. 720 (2)
147. 1115 (1)
148. 289 (2)
149. 4.5 (2)
150. 54 (3)

WORKOUT 2
151. 1.64 (3)
152. 1* (3)
153. 9.41 (2)
154. 125 (3)
155. 216,000 (3)
156. 12 (4)
157. 14.25 (2)
158. 504 (2)
159. 7.5 (3)
160. 7 (3)

WORKOUT 3
161. 1.75 (2)
162. 35 (1)
163. 13.9 (4)
164. 160 (2)
165. 12 (4)
166. 10,101 (3)
167. 326,000 (4)
168. 33 (1)
169. 70 (4)
170. 1.2 (3)

WORKOUT 4
171. −4/7 (3)
172. 5760 (3)
173. 1 (4)
174. 84 (3)
175. 9 (3)
176. 24 (5)
177. 10.3 (3)
178. 28 (3)
179. −17/4 (4)
180. 14 (4)

WORKOUT 5
181. 94 (4)
182. 64 (3)
183. 79 (3)
184. 10.3 (5)
185. 108 (4)
186. 990 or 990.00 (4)
187. 21 (5)
188. 245 (3)
189. 5/2 (4)
190. 20 (2)

WORKOUT 6
191. 25 (5)
192. 16 (4)
193. 3.9 (2)
194. 1.5 (4)
195. 30 (4)
196. 46.2 (6)
197. 8 (3)
198. 28.1 (4)
199. 0.75 (4)
200. $2\sqrt{2}$ (5)

** The plural form of the units is always provided in the answer blank, even if the answer appears to require the singular form of the units.*

PART 2

Warm-Up 12

201.__________ If $4^x = 16^{x-1}$, what is the value of x?

202.__________ % In Cycletown, it rains every Monday, Wednesday and Saturday and is sunny all other days of the week. Based on the calendar shown, on what percent of the days in April 2021 will it be sunny in Cycletown?

2021
APRIL

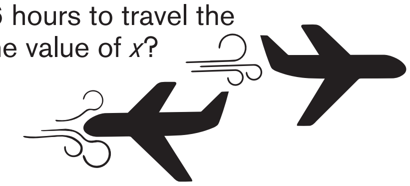
Su	Mo	Tu	We	Th	Fr	Sa
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

203.__________ ordered triples How many ordered triples of nonnegative integers (a, b, c) satisfy $a + b + c = 7$?

204.__________ in^2 Two sides of a triangle have lengths 7 inches and 12 inches. What is the greatest possible area of the triangle, in square inches?

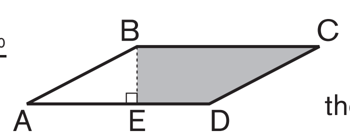
205.__________ What is the value of $\dfrac{4^{26}+8^{15}}{32^{9}+16^{11}}$?

206.__________ An airplane traveling east has a tailwind, and its speed is $550 + x$ mi/h. When the airplane flies west, it has a headwind and travels at $550 - x$ mi/h. If it takes 5 hours to travel from the West Coast to the East Coast of the United States and 6 hours to travel the same path from the East Coast to the West Coast, what is the value of x?



207.__________ ways Mrs. Frizzle has nine students in her biology class. In how many ways can she assign her students to lab groups of two or three students?

208.__________ %



In parallelogram ABCD, shown here, with altitude BE, BC = 10, AE:DE = 3:2 and AE:BE = 2:1. What percent of the figure is shaded trapezoid BCDE?

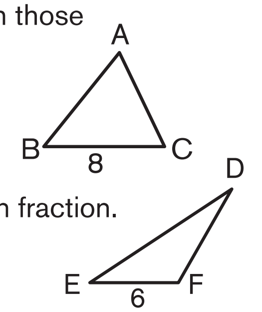
209.__________ If $f(x) = (x + 1)^2$ and $g(x) = 2[f(x)] - 4$, what is the sum of all values of x for which $f(x) = g(x)$?

210.__________ zeros When 15! is calculated, how many zeros appear to the right of the last nonzero digit?

Warm-Up 13

211.__________ What is the absolute difference of $\frac{1}{3}$ and 0.33333? Express your answer as a common fraction.

212.__________ license plates The license plates in Eulerville are made of seven distinct characters, six of which are letters from A to Z inclusive and one of which is a digit from 0 to 9 inclusive. How many different Eulerville license plates include the word "MATH" with those four letters consecutive and in that order?



213.__________ The two triangles shown have the same vertical altitude. What is the ratio of the area of $\triangle ABC$ to that of $\triangle DEF$? Express your answer as a common fraction.

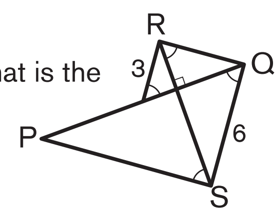
214.__________ bicycles

Happy Trails Cycles has a collection of unicycles, bicycles and tricycles in stock, with at least one of each item in inventory. If there are a total of 73 wheels on the collection of unicycles, bicycles and tricycles, what is the maximum possible number of bicycles in the shop's inventory?

215.__________ If $(x + 5) + (2x + 5) + (3x + 5) + \cdots + (98x + 5) + (99x + 5) + (100x + 5) = 0$, what is the value of x? Express your answer as a common fraction.

216.__________ If 50^k is a divisor of 50!, what is the greatest possible integer value of k?

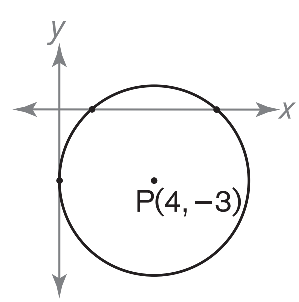
217.__________ units In the figure shown, the marked angles are congruent. If $\overline{PQ} \perp \overline{RS}$, what is the length of $\overline{PQ}$? Express your answer in simplest radical form.



218.__________ For what value of a, where $a > 0$, is it true that $a^{\frac{5}{3}} = \frac{5}{3}a$? Express your answer as a common fraction in simplest radical form.

219.__________

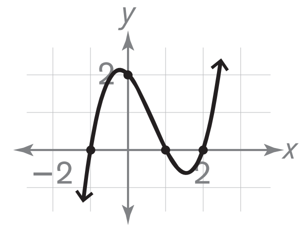
Shavonne planted a dogwood tree. It was A feet B inches tall, where A and B are both positive integers and $B < 12$. Four years later, she measured the tree again. She noticed its height had increased by exactly 125% and it was now B feet A inches tall. What is the value of $A + B$?



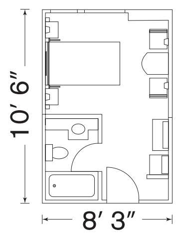
220.__________ A circle with center P(4, −3) is tangent to the y-axis as shown. What is the product of the x-coordinates of the two points where the circle intersects the x-axis?

Warm-Up 14

221. ______ The figure shows the graph of $y = ax^3 + bx^2 + cx + d$ for real numbers a, b, c and d, with its x- and y-intercepts indicated. What is the value of $a + b + c + d$?



222. ______ people In a survey of 200 people, 161 reported owning a car, 58 owned a bicycle and 74 had a valid mass transit pass. A total of 12% of the survey respondents reported owning all three, while 2.5% reported owning none. How many of the people surveyed owned exactly one of the three?



223. ______ yd² A rectangular room measures 10 feet 6 inches by 8 feet 3 inches. What is its area in square yards? Express your answer as a mixed number.

224. ______ amounts Dakota has four 49¢ stamps and five 21¢ stamps. How many different monetary amounts of postage can Dakota put on an envelope, using one or more of these stamps?

225. ______ Let j be a number between 100 and 150, and let k be a number between 400 and 550. Now suppose $n = 2j$, $p = n + k$, $q = |n - k|$, $r = pq$, $s = k^2$, $t = |r - s|$ and $u = \frac{t}{j}$. What is the ratio of n to u? Express your answer as a common fraction.

226. ______ units² What is the area of the hexagon with consecutive vertices at (0, 0), (3, −4), (8, −3), (5, 1), (6, 6) and (1, 5)?

227. ______ In a 10 by 10 multiplication chart with columns and rows both numbered 1 through 10, the number in each cell in the chart is the product of the row and column numbers. What is the arithmetic mean of the numbers in all 100 cells of this chart? Express your answer as a decimal to the nearest hundredth.

228. ______ Five fair coins are flipped. Those that land heads up are flipped again. After the second round of flips, what is the probability that all five coins are now tails up? Express your answer as a common fraction.

229. ______ Ariceli walks x_1 yards from home, then turns right and walks x_2 yards, then turns right again and walks x_3 yards, and continues in this way until she has turned 99 times and then walked x_{100} yards. She ends up exactly d yards from home. If $x_1, x_2, \ldots, x_{100}$ are positive consecutive odd integers in increasing order, what is the value of d? Express your answer in simplest radical form.

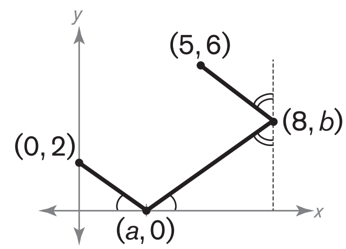
230. ______ units What is the radius of a circle whose equation is $x^2 + y^2 + 6x + 4y - 12 = 0$?

Workout 7

231.__________ If the digits 1 through 5 are randomly selected with replacement to create a four-digit number, what is the probability the number is divisible by 15? Express your answer as a common fraction.

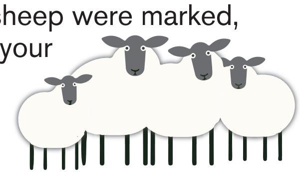
232.__________ If x is a positive number and $x = \sqrt{3+\sqrt{3+\sqrt{3+\cdots}}}$, what is the value of x? Express your answer as a decimal to the nearest tenth.

233.__________ In the figure, two pairs of congruent angles are marked. What is the value of a? Express your answer as a common fraction.



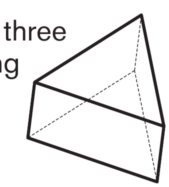
234.__________ Tye is adding consecutive perfect squares $1^2 + 2^2 + 3^2 + \cdots$. After reaching a sum of 10,000, Tye realizes that he must have skipped two perfect squares, but he doesn't know which ones. What is the least possible sum of the perfect squares Tye skipped?

235.__________ sheep One day, 91 randomly chosen sheep on a ranch were herded into a pen, marked and released into the pasture. Two days later, 53 randomly chosen sheep were herded into a pen, and the marked sheep were counted. If 3 of the 53 herded sheep were marked, what is the expected total number of sheep on the ranch? Express your answer to the nearest whole number.



236.__________ subsets How many subsets of one or more elements of the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} contain the element 1 or the element 2 or both?

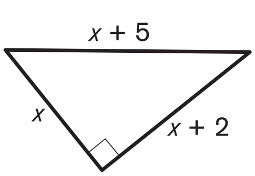
237.__________ ways Each face of a right prism with equilateral triangle bases can be painted one of three colors: red, green or black. Two colorings are considered the same if, after being colored one way, the prism can be rotated, flipped or both to obtain the other coloring. How many different ways are there to paint the prism?



238.__________ What is the sum of the positive integers less than 100 that cannot be expressed as the sum of two or more consecutive positive integers?

239.__________ % Todd has bins containing marbles. Of these bins, 17.11% contain 3 marbles each, all of them red. Each of the remaining bins contains 2 marbles each, both of them blue. If Todd dumps all the marbles into a large bucket, what percent of the marbles in the bucket will be blue? Express your answer to the nearest whole percent.

240.__________ units



The right triangle shown has side lengths x, $x + 2$ and $x + 5$. What is the value of x? Express your answer in simplest radical form.

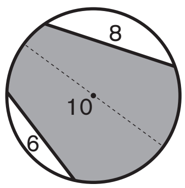
Workout 8

241.__________ A galleon is worth 17 sickles, and a sickle is worth 29 knuts. Suppose Ron has twice as many sickles as knuts and three times as many galleons as knuts. The total value of all his coins is N galleons, where N is an integer. What is the least possible value of N?

242.__________ words Using the standard 26-letter alphabet, how many five-letter "words" can be made with A, E, I, O or U as the initial letter but not as the final letter?

243.__________ divisors How many positive integer divisors does 10! have?

244.__________ units²



The circle shown has a diameter of 10 units. What is the area of the shaded region bounded by chords of lengths 6 units and 8 units? Express your answer as a decimal to the nearest tenth.

245.__________ % The Avondale Anteaters and the Bassburg Bears will be playing a best-of-five baseball series. Once a team has won three games, no more games will be played in the series. Based on previous performance, the predicted probability of the Anteaters winning any particular game against the Bears is 60%. What is the probability that the series between the Anteaters and Bears will consist of exactly 4 games? Express your answer to the nearest whole percent.

246.__________ % A regulation soccer field is rectangular with length of 100 to 110 meters, inclusive, and width of 64 to 73 meters, inclusive. A particular soccer field is the smallest-size regulation field, but it is being expanded to be the largest-size regulation field. What will be the percent increase in its area? Express your answer to the nearest whole percent.

247.__________ units² What is the area of a circle inscribed in a triangle with vertices at (1, 6), (2, 5) and (0, 4)? Express your answer as a decimal to the nearest hundredth.

248.__________ What is the base-ten value of the product $351_8 \times 10_8$?

249.__________ If $f(x) = ax^2 + bx + c$ and $f(2) = 3$, $f(5) = -6$ and $f(7) = 10$, what is the value of c?

250.__________ cm² Joy starts with a blank canvas. She marks the center point O and then marks points R, L, A and B exactly 10 cm to the right of, left of, above and below point O, respectively. Joy paints blue any part of the canvas within 10 cm of R. She paints green any part within 10 cm of L. She paints orange any part within 10 cm of A. She paints yellow any part within 10 cm of B. What is the total area of the canvas that she paints with at least two colors? Express your answer to the nearest whole number.

SOLUTIONS

The solutions provided here are only *possible* solutions. It is very likely that you or your students will come up with additional—and perhaps more elegant—solutions. Happy solving!

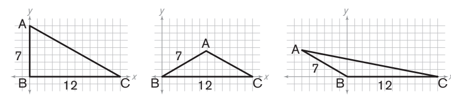
Warm-Up 12

201. Since $16 = 4^2$, we can rewrite the equation as $4^x = (4^2)^{x-1} \rightarrow 4^x = 4^{2x-2}$. Now that both powers have the same base, we can set the exponents equal to each other to get $x = 2x - 2$. So, $x =$ **2**.

202. During April 2021, there are four Mondays, four Wednesdays and four Saturdays. So, in Cycletown, it will rain on 4 × 3 = 12 days that month, meaning it will be sunny on 30 − 12 = 18 days. Therefore, the percent of the days that it will be sunny is 18/30 = 0.6 = **60**%.

0	0	0	0	1	1	1	2
0	1	2	3	1	2	3	2
7	6	5	4	5	4	3	3
3	6	6	6	3	6	3	3

203. The table shows the eight combinations of three nonnegative integers that have a sum of 7, and the number of ways each can be ordered. So, the number of ordered triples meeting the given criteria is 4 × 3 + 4 × 6 = 12 + 24 = **36** ordered triples. Alternatively, if $a = 0$, then there are 8 possible values for b that will allow $a + b + c = 7$, namely 0 through 7. If $a = 1$, then there are 7 possible values of b: 0 through 6. Continue this pattern, and for $a = 7$, there is 1 possible value of b, namely 0. So, in all there are 8 + 7 + 6 + ⋯ + 1 = 9 × 8 ÷ 2 = **36** ordered triples.



204. If we think of the side of length 12 inches as the base, then the maximum area occurs when the height is greatest. This occurs when the side of length 7 inches forms an altitude to the base; otherwise the height will be less, as illustrated in the figures shown here. Therefore, the greatest possible area is 1/2 × 12 × 7 = **42** in^2.

205. Recall that $4 = 2^2$, $8 = 2^3$, $16 = 2^4$ and $32 = 2^5$. We can rewrite the expression so that all powers have a base of 2. Doing so, we get $[(2^2)^{26} + (2^3)^{15}]/[(2^5)^9 + (2^4)^{11}] = (2^{52} + 2^{45})/(2^{45} + 2^{44}) = 2^{44}(2^8 + 2)/2^{44}(2 + 1) = (256 + 2)/3 = 258/3 =$ **86**.

206. The distance traveled is the same in both directions. So, since Distance = Rate × Time we can write $5(550 + x) = 6(550 - x)$. Simplifying and solving for x, we get $5 \times 550 + 5x = 6 \times 550 - 6x \rightarrow 11x = 6 \times 550 - 5 \times 550 \rightarrow 11x = 550 \rightarrow x =$ **50**, meaning the wind speed is 50 mi/h.

207. Mrs. Frizzle can split the 9 people into three groups of 3 or into one group of 3 and three groups of 2. First, let's look at how to assign students to three groups of 3. There are ${}_9C_3$ = 9!/(6! × 3!) = (9 × 8 × 7)/(3 × 2 × 1) = 84 ways to assign students to the first group of 3. There are ${}_6C_3$ = 6!/(3! × 3!) = (6 × 5 × 4)/(3 × 2 × 1) = 20 ways to assign students to the second group of 3. There is only 1 way to assign the remaining students to the final group of 3. But since there are 3! = 3 × 2 × 1 = 6 orders in which the three groups can be created, there are actually 84 × 20 × 1 ÷ 6 = 14 × 20 = 280 ways to assign students to three groups of 3. Now, let's look at how to assign students to one group of 3 and three groups of 2. There are ${}_9C_3$ = 84 ways to assign students to the group of 3. There are ${}_6C_2$ = 6!/(4! × 2!) = (6 × 5)/(2 × 1) = 15 ways to assign students to the first group of 2. There are ${}_4C_2$ = 4!/(2! × 2!) = (4 × 3)/(2 × 1) = 6 ways to assign students to the second group of 2. There is only 1 way to assign the remaining students to the last group of 2. Again, we must account for the 6 orders in which the three groups of 2 can be created. That means there are 84 × 15 × 6 × 1 ÷ 6 = 84 × 15 = 1260 ways to assign the students to one group of 3 and three groups of 2. In all, Mrs. Frizzle can assign her students to lab groups in 280 + 1260 = **1540** ways.

208. Since ABCD is a parallelogram, side AD must equal side BC, so it's also 10 units long. If we split 10 units in the ratio 3:2, we get AE = 6 and DE = 4. We know the ratio of AE to BE is 2 to 1, so BE = 3. The total area of the parallelogram is 10 × 3 = 30 units^2, and the area of △AEB is 1/2 × 6 × 3 = 9 units^2. The area of the shaded region is the total area minus the unshaded area, or 30 − 9 = 21 units^2. This is 21/30 = 7/10 = **70**% of the figure.

209. Let $u = x + 1$. So, $f(x) = u^2$ and $g(x) = 2[f(x)] - 4 = 2u^2 - 4$. We are asked to find the sum of the values of x for which $f(x) = g(x)$. Setting these expressions equal to each other and solving for u yields $u^2 = 2u^2 - 4$, so $u^2 = 4$ and $u = \pm 2$. Substituting into the expression for u, we see that the functions are equal when $2 = x + 1$, so $x = 1$, and when $-2 = x + 1$, so $x = -3$. The sum of these values is 1 + (−3) = **−2**.

210. We don't actually need to calculate 15! = 15 × 14 × 13 × ⋯ × 2 × 1 to determine how many zeros appear to the right of the last nonzero digit. First, we note that every zero at the end of the number corresponds to an instance of 10 as a factor. Since 10 = 2 × 5, we need to consider how many times 2 and 5 occur as factors of 15!. Next, we note that 2 occurs as a factor of 15! many times but 5 occurs only three times—once each in the numbers 5, 10 and 15. When each of the three instances of 5 as a factor is paired with one of the instances of 2, the result is an instance of 10 as a factor of 15!. Therefore, 15! has **3** zeros to the right of the last nonzero digit.

Warm-Up 13

211. To determine the absolute difference of 1/3 and 0.33333, we'll rewrite the decimal in fraction form. We have |1/3 − 33,333/100,000| = 100,000/300,000 − 99,999/300,000 = **1/300,000**.

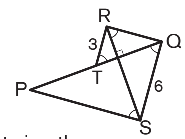
212. If we consider the word "MATH" as a single block, we still need two more letters and one digit to complete the seven characters of an Eulerville license plate. We are told that the seven characters must be distinct, so there are ${}_{22}C_2$ = 22!/(20! × 2!) = (22 × 21)/(2 × 1) = 231 ways to choose the two remaining letters, and there are 10 ways to choose a digit from 0 to 9, inclusive. That means there are 231 × 10 = 2310 ways to choose the remaining three characters. The word "MATH" and the three other characters can be arranged in 4! = 4 × 3 × 2 × 1 = 24 ways. So, the number of possible MATH license plates in Eulerville is 2310 × 24 = **55,440** license plates.

213. Since △ABC and △DEF have the same vertical altitude, the ratio of their areas will be the ratio of the lengths of their bases, namely 8/6 = **4/3**.

214. We know there are 73 tires on the unicycles, bicycles and tricycles in inventory. To maximize the number of bicycles, we need to minimize the numbers of unicycles and tricycles. Assuming there is only one of each accounts for 1 + 3 = 4 tires and leaves 73 − 4 = 69 tires. This could be from 1 unicycle and the maximum of (69 − 1) ÷ 2 = 68 ÷ 2 = **34** bicycles.

215. The equation can be rewritten as $x(1 + 2 + 3 + \cdots + 98 + 99 + 100) + 5 \times 100 = 0$. The sum of the first 100 positive integers is (1 + 100) × (100 ÷ 2) = 101 × 50. So, we have $x \times 101 \times 50 + 500 = 0$. Simplifying and solving for x, we get $50(101x + 10) = 0 \to 101x + 10 = 0 \to 101x = -10 \to x =$ **−10/101**. Alternatively, if the sum of this arithmetic sequence is to equal 0, then the first 50 terms must be the additive inverses of the last 50 terms, in reverse order. In particular, $x + 5 = -(100x + 5)$. This equation simplifies to $101x = -10$, so $x =$ **−10/101**.

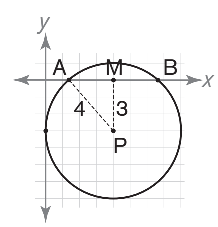
216. Recall that 50! = 50 × 49 × 48 × ⋯ × 2 × 1 and that $50 = 2 \times 5^2$. We need to determine the number of combinations of 2×5^2 that occur in this product. Noting that there are far more instances of 2 as a factor of this product, we'll focus on the number of instances of 5^2, since we are guaranteed that there will be an instance of 2 to pair with each one. The numbers in this product that have 5 as one or more factors are 5, 10, 15, 20, 25, 30, 35, 40, 45 and 50. Since 5 appears as a factor 12 times in these numbers, that gives us a total of 12 ÷ 2 = 6 instances of 5^2, and thus 6 instances of $2 \times 5^2 = 50$ as a factor. Therefore, the greatest integer value of k for which 50^k is a divisor of 50! is k = **6**.



217. We can use properties of similar triangles to solve for some missing side lengths. There are many similar triangles in the figure, including triangles RQT and QSR. We can write the proportion 3/RQ = RQ/6. Cross-multiplying, we see that $RQ^2 = 3 \times 6$, so $RQ^2 = 18$ and $RQ = \sqrt{18} = 3\sqrt{2}$ units. Based on the Pythagorean theorem, we know that $QT = \sqrt{[3^2 + (3\sqrt{2})^2]} = \sqrt{(9 + 18)} = \sqrt{27} = 3\sqrt{3}$ units. Similar triangles SPQ and RQT have sides in the ratio QS/RT = 6/3 = 2/1. That means each side of △SPQ is twice the length of the corresponding side of △RQT. Therefore, since $QT = 3\sqrt{3}$ units, corresponding side PQ must have length $2 \times 3\sqrt{3} =$ **$6\sqrt{3}$** units.

218. If we divide both sides of the equation by a, then by properties of exponents, we get $a^{2/3} = 5/3$, which can be rewritten as $\sqrt[3]{a^2} = 5/3$. Then, cubing both sides of this equation yields $a^2 = 5^3/3^3$. Finally, since we know that $a > 0$, taking the square root of both sides gives us $a = \sqrt{(5^3/3^3)} = \sqrt{5^3}/\sqrt{3^3} = 5\sqrt{5}/(3\sqrt{3}) \times \sqrt{3}/\sqrt{3} =$ **$5\sqrt{15}/9$**.

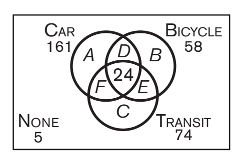
219. In inches, the tree's original height was $12A + B$, and its new height is $12B + A$. The increase in height of 125% means that the new height is 100 + 125 = 225% of the original height. In other words, four years later the tree was 225/100 = 9/4 its original height. Given this, we can write the equation $9/4(12A + B) = 12B + A$. Simplifying, we get $27A + (9/4)B = 12B + A \to 26A = (39/4)B \to 104A = 39B \to A/B = 39/104 = 3/8$. Since A and B must be integers with $B < 12$, the only possible solution with A and B in this ratio is $A = 3$ and $B = 8$. So, the value of $A + B$ is 3 + 8 = **11**.



220. Let A and B be the two points where the circle intersects the x-axis, as shown. The midpoint of chord AB is M(4, 0). Right triangle AMP has hypotenuse AP, which is a radius of the circle of length 4, and leg MP of length 3. Using the Pythagorean theorem, we determine that leg AM has length $\sqrt{(4^2 - 3^2)} = \sqrt{(16 - 9)} = \sqrt{7}$. Therefore, the circle intersects the x-axis at $A(4 - \sqrt{7}, 0)$ and $B(4 + \sqrt{7}, 0)$. The product of the x-coordinates of these points is $(4 - \sqrt{7}) \times (4 + \sqrt{7}) = 16 - 7 =$ **9**.

Warm-Up 14

221. From the graph, we see that $x = -1$, $x = 1$ and $x = 2$ are solutions to the equation $ax^3 + bx^2 + cx + d = 0$. That means that $(x + 1)(x - 1)(x - 2) = 0$. Multiplying, we get $(x^2 - 1)(x - 2) = 0 \to x^3 - 2x^2 - x + 2 = 0$. Looking at the graph, we see that the y-intercept is at (0, 2), so $d = 2$, which agrees with the expanded cubic equation. So, we have $y = x^3 - 2x^2 - x + 2$, meaning $a = 1$, $b = -2$, $c = -1$ and $d = 2$. The value of $a + b + c + d$, then, is 1 + (−2) + (−1) + 2 = **0**. Alternatively, we can get the value of $a + b + c + d$ by substituting $x = 1$ into the polynomial. Because the graph shows that the corresponding y value at $x = 1$ is 0, it must be that $a + b + c + d =$ **0**.

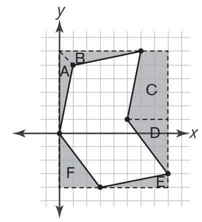


222. Of the 200 survey respondents, 12%, or 200 × 0.12 = 24 people, reported owning a car, a bicycle and a valid mass transit pass, while 2.5%, or 200 × 0.025 = 5 people, reported owning none of these. The Venn diagram shown summarizes what we know about the survey respondents. We let A, B and C represent the numbers of people who own a car, bicycle and mass transit pass, respectively. Our ultimate goal is to find $A + B + C$, which is the number of people who own exactly one of the three. Next, we let D, E and F represent people who own a car and a bicycle, a bicycle and a mass transit pass, and a car and a mass transit pass, respectively. Combined, the 161 car owners, 58 bicycle owners and 74 mass transit pass owners equal 161 + 58 + 74 = 293 people. This total, however, counts the intersection of each pair of groups twice, and counts the intersection of all three groups three times. So, we have $A + B + C + 2(D + E + F) + 72 = 293$, which simplifies to $A + B + C + 2(D + E + F) = 221$. We also know that $A + B + C + D + E + F + 24 + 5 = 200$, which simplifies to $A + B + C + D + E + F = 171$. The difference between these two equations is $D + E + F = 221 - 171 = 50$. Substituting this back into $A + B + C + D + E + F = 171$, we find that $A + B + C + 50 = 171$, so $A + B + C = 121$. Therefore, the number of people who own exactly one of the three is **121** people.

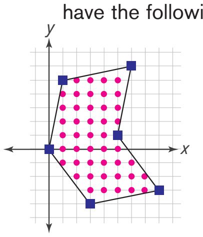
223. We are told that the room has dimensions 10 feet 6 inches, or 21/2 feet, by 8 feet 3 inches, or 33/4 feet. In yards, that's 21/2 × 1/3 = 7/2 yards by 33/4 × 1/3 = 11/4 yards. Thus, the area of this room, in square yards, is 7/2 × 11/4 = 77/8 = $\mathbf{9\frac{5}{8}}$ yd^2.

224. We need to be careful in our counting here, since 49 and 21 have a common factor of 7. It would take seven of the 21¢ stamps to equal three of the 49¢ stamps, and Dakota only has five of the 21¢ stamps. With the danger of a duplicate amount of postage set aside, we reason as follows: Dakota can use from 0 to 4 of the 49¢ stamps, yielding 5 different choices. Dakota can use from 0 to 5 of the 21¢ stamps, yielding 6 different choices. For each of the 5 choices for how many 49¢ stamps to use, there are 6 choices for how many 21¢ stamps to use. That gives us 5 × 6 = 30 outcomes, one of which combines zero 49¢ stamps and zero 21¢ stamps. Therefore, using these stamps, Dakota can apply postage in 30 − 1 = **29** amounts.

225. Since $n = 2j$ and $u = t/j$, and we are asked to find n/u, let's see if we can get both values in terms of a single variable. Given that $n = 2j$, it follows that $p = n + k = 2j + k$, and $q = |n - k| = |2j - k| = k - 2j$, since $200 < 2j < 300$ and $400 < k < 550$. Next, we have $r = pq = (2j + k)(k - 2j) = k^2 - 4j^2$. Then, since $s = k^2$, it follows that $t = |r - s| = |k^2 - 4j^2 - k^2| = 4j^2$. Finally, we have $u = t/j = 4j^2/j = 4j$. We now see that the desired ratio is $n/u = 2j/(4j) =$ **1/2**.

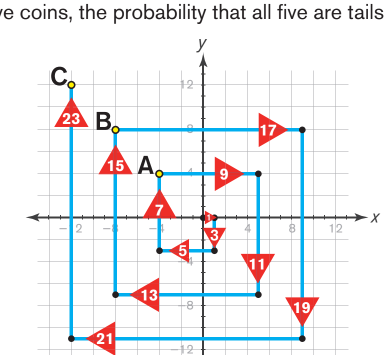


226. We circumscribe the hexagon with an 8-by-10 rectangle of area 8 × 10 = 80 units^2, as shown in the figure on the right. The shaded region inside the rectangle that surrounds the hexagon can be divided into the six regions labeled A through F. The regions have the following areas: A = B = 1/2 × 6 × 1 = 3 units^2, C = 1/2 × (2 + 3) × 5 = 12.5 units^2, D = F = 1/2 × 4 × 3 = 6 units^2, E = 1/2 × 5 × 1 = 2.5 units^2. The area of the hexagon, then, is 80 − (2 × 3 + 12.5 + 2 × 6 + 2.5) = 80 − (6 + 12 + 15) = 80 − 33 = **47** units^2. Alternatively, Pick's theorem says that for a simple ploygon whose vertices have integer coordinates, the area can be calculated from the number of interior lattice points i and the number of boundary lattice points b using the formula $A = i + b/2 - 1$. In the figure on the left, ● and ■ are used to denote interior and boundary lattice points, respectively. There are 45 interior lattice points and 6 boundary lattice points. So, we have $i = 45$, $b = 6$ and $A = 45 + 6/2 - 1 = 45 + 3 - 1 =$ **47** units^2. This problem can also be solved by the shoelace method (See *2018-2019 MATHCOUNTS School Handbook*, p. 58, for a detailed explanation of this method). In this case, we have $A = [(0 \times (-4) + 3 \times (-3) + 8 \times 1 + 5 \times 6 + 6 \times 5 + 1 \times 0) - (0 \times 3 + (-4) \times 8 + (-3) \times 5 + 1 \times 6 + 6 \times 1 + 5 \times 0)]/2 = [59 - (-35)]/2 = 94/2 =$ **47** units^2.



227. The sum of the numbers in row 1 is 1 + 2 + 3 + ⋯ + 9 + 10 = 55. The sum of the numbers in row 2 are 2 + 4 + 6 + ⋯ + 18 + 20 = 2(1 + 2 + 3 + ⋯ + 9 + 10) = 2 × 55. Similarly, the sum of the numbers in row 3 is 3(1 + 2 + 3 + ⋯ + 9 + 10) = 3 × 55. In fact, the sum of the totals of all ten rows is 1 × 55 + 2 × 55 + 3 × 55 + ⋯ + 9 × 55 + 10 × 55 = (1 + 2 + 3 + ⋯ + 9 + 10) × 55 = 55 × 55 = 3025. The arithmetic mean of these 100 numbers is 3025 ÷ 100 = **30.25**.

228. There are six ways for all five coins to end up with tails after two flips. The probability that all five coins land heads up and then all five land tails up is 1/32 × 1/32 = <u>1/1024</u>. The probability that four coins land heads up and then all four land tails up is 5 × 1/32 × 1/16 = <u>5/512</u>. The probability that three coins land heads up and then all three land tails up is 10 × 1/32 × 1/8 = <u>10/256</u>. The probability that two coins land heads up and then both land tails up is 10 × 1/32 × 1/4 = <u>10/128</u>. The probability that one coin lands heads up and then lands tails up is 5 × 1/32 × 1/2 = <u>5/64</u>. The probability that all five coins land tails up on the first flip is <u>1/32</u>. The probability that all five coins are tails up after the second flip, then, is 1/1024 + 5/512 + 10/256 + 10/128 + 5/64 + 1/32 = 1/1024 + 10/1024 + 40/1024 + 80/1024 + 80/1024 + 32/1024 = **243/1024**. Alternatively, a much simpler solution is possible if you recognize that each coin gets two chances to land tails up, and the only way for it *not* to land tails up is for it to land heads up and then land heads up again on the second flip. The probability of this happening is 1/2 × 1/2 = 1/4. Therefore, each coin has a 1 − 1/4 = 3/4 chance of being tails up after the second flip (if necessary). Because there are five coins, the probability that all five are tails up is $(3/4)^5 = 3^5/4^5 =$ **243/1024**.



229. Let's imagine that Ariceli is walking on the coordinate grid. Suppose she starts at the origin and walks to the right along the x-axis for 1 unit, then down 3 units, then to the left 5 units, then up 7 units and so forth, as shown. Since 100 is a multiple of 4, we want to look at the places where Ariceli will be after walking x_n yards when n is a multiple of 4. The locations A, B and C in the figure are the places where Ariceli will be after walking x_4, x_8 and x_{12} yards, respectively. We have $A(-4, 4)$, $B(-8, 8)$ and $C(-12, 12)$, and if the pattern continues, Ariceli will be at the point $(-100, 100)$ after walking x_{100} yards. So, $d = \sqrt{(100^2 + 100^2)} = \sqrt{(2 \times 100^2)} = \mathbf{100\sqrt{2}}$. It should be noted that although the problem does not state the first odd integer distance walked was 1 unit, it doesn't matter what odd integer distance she starts with (walking to the right), because each sequence of four walks of consecutive odd numbers of units moves her 4 units to the left and 4 units up.

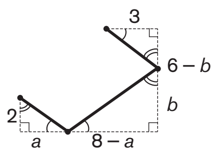
230. We are given the equation $x^2 + y^2 + 6x + 4y - 12 = 0$. The standard form of the equation for a circle centered at (h, k) with radius r is $(x - h)^2 + (y - k)^2 = r^2$. Using a technique called completing the square, we will rewrite the equation in this form to reveal the radius. Doing so, we get $x^2 + 6x + y^2 + 4y = 12 \rightarrow (x^2 + 6x + 9) + (y^2 + 4y + 4) = 12 + 13 \rightarrow (x + 3)^2 + (y + 2)^2 = 25$. We now have an equation for the circle in standard form, where $r^2 = 25$, and $r = 5$. We see that the given circle has center $(-3, -2)$ and radius **5** units.

Workout 7

231. Since replacement is allowed, there are 5 available digits to choose from for each place value in the four-digit number. That's a total of 5 × 5 × 5 × 5 = 625 outcomes. For a number to be divisible by 15, it must be divisible by 3 and by 5. For divisibility by 5, the units digit can only be 5. For divisibility by 3, the sum of the digits must be divisible by 3. A units digit of 5 is 2 more than a multiple of 3, so the remaining digits must have a sum that is 1 more than a multiple of 3. The table shows the possible sets of digits with the number of ways each can be rearranged. The probability is thus **42/625**.

Digits	Ways
112	3
115	3
124	6
133	3
145	6
223	3
235	6
244	3
334	3
355	3
445	3

232. Squaring both sides of the equation yields $x^2 = 3+\sqrt{3+\sqrt{3+\cdots}}$. Notice that the expression under the radical is the original expression for x. That means $x^2 = 3 + x \rightarrow x^2 - x - 3 = 0$. We can solve this equation using the quadratic formula $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$. Since the coefficients of the quadratic equation are $a = 1$, $b = -1$ and $c = -3$, we have $x = \frac{-(-1)\pm\sqrt{(-1)^2-4\times1\times(-3)}}{2\times1} = \frac{1\pm\sqrt{13}}{2}$. We are told that x is positive, so it follows that $x = \frac{1+\sqrt{13}}{2} \approx$ **2.3**.



233. With the addition of a few line segments, we can create three similar right triangles, as shown. Some side lengths have been labeled based on the known coordinates. By properties of similar figures, we can write the proportion $2/a = b/(8 - a)$. Cross-multiplying, we get $2(8 - a) = ab \rightarrow 16 - 2a = ab$. We can also write the proportion $2/a = (6 - b)/3$. Cross-multiplying, we get $2 \times 3 = a(6 - b) \rightarrow 6 = 6a - ab \rightarrow ab = 6a - 6$. Setting the two expressions for ab equal to each other yields $16 - 2a = 6a - 6$. Simplifying and solving for a, we get $8a = 22$, so $a = 22/8 =$ **11/4**.

234. The sum of the first n squares can be found by using the formula $S_n = n(n + 1)(2n + 1)/6$. The least value of n for which S_n exceeds 10,000 is $n = 31$, with $S_{31} = (31 \times 32 \times 63)/6 = 10{,}416$. Since Tye's sum was exactly 10,000, the sum of the squares he skipped must have been 10,416 – 10,000 = **416**, and we confirm that $416 = 2^2 + 20^2$.

235. In this "capture-recapture" scenario, we reason that the 3 out of 53 sheep must be proportional to 91 out of x, the unknown total number of sheep. So we set up the proportion $3/53 = 91/x$. Cross-multiplying and solving for x, we get $3x = 53 \times 91 \rightarrow x = 53 \times 91 \div 3 \approx 1608$. Thus, the expected total number is **1608** sheep.

236. The total number of subsets of one or more elements of the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} is the sum of the numbers of subsets with 1, 2, 3, …, 10 elements. That's ${}_{10}C_1 + {}_{10}C_2 + {}_{10}C_3 + \cdots + {}_{10}C_{10}$. This is the sum of the entries in row 10 of Pascal's triangle, which is 2^{10}. Now let's consider the set {3, 4, 5, 6, 7, 8, 9, 10}, which excludes the elements 1 and 2. The sum of the numbers of subsets with 1, 2, 3, …, 8 elements is ${}_8C_1 + {}_8C_2 + {}_8C_3 + \cdots + {}_8C_8$. This is the sum of the entries in row 8 of Pascal's triangle, which is 2^8. Thus, the number of subsets of the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} that contain 1 or 2 or both is $2^{10} - 2^8 =$ **768** subsets.

237. First, let's consider the equilateral triangle bases of the prism. These bases could be all painted the same color in 3 ways: both red (RR), both green (GG), both black (BB). They can be painted with two different colors in 3 ways: RG, RB, GB. That's 6 possibilities for the triangular bases. Note that we don't consider GR, BR or BG to be different, since the prism could just be flipped. Next, we consider the three rectangular faces. These rectangular faces can be all the same color in 3 ways: RRR, GGG, BBB. They could be painted with two colors in 6 ways: RRG, RRB, GGR, GGB, BBR, BBG. They can be painted with three colors in 2 ways: RGB and RBG. That's 11 possibilities for the rectangular faces. Combining the possibilities for the bases with the possibilities for the rectangles, there would seem to be 6 × 11 = 66 ways to paint the prism, but we have duplicates. Note that RGB and RBG are mirror images of each other when the bases are different, but they can be flipped and superimposed when the bases are the same. Therefore, the total number of ways to paint the prism is 6 × 11 – 3 = **63** ways.

238. A property of odd numbers is that they all can be expressed as the sum of two consecutive integers: $n + (n + 1) = 2n + 1$. A property of multiples of 3 is that they can be written as a sum of three consecutive integers: $(n - 1) + n + (n + 1) = 3n$, for $n > 1$. A property of multiples of 5 is that they can be written as a sum of five consecutive integers: $(n - 2) + (n - 1) + n + (n + 1) + (n + 2) = 5n$, for $n > 2$. There are similar properties for multiples of every odd number. In fact, the only numbers that cannot be written as a sum of two or more consecutive positive integers are numbers that don't have any odd factors, namely the powers of 2. The powers of 2 less than 100 are 1, 2, 4, 8, 16, 32 and 64. The sum of these numbers is **127**.

239. Without loss of generality, we can suppose that Todd has 10,000 bins. Since 17.11% of 10,000 is 1711, Todd has 1711 bins with only 3 red marbles each. So, there are 1711 × 3 = 5133 red marbles. The remaining 10,000 – 1711 = 8289 bins contain 2 blue marbles each. So, there are 8289 × 2 = 16,578 blue marbles. If Todd dumps all the marbles into a large bucket, there will be 5133 + 16,578 = 21,711 marbles in the bucket. The blue marbles will represent 16,578/21,711 ≈ 0.76 = **76**% of the marbles.

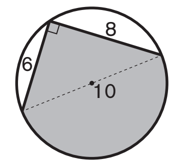
240. We can use the Pythagorean theorem to set up an equation and solve for x. We have $x^2 + (x + 2)^2 = (x + 5)^2 \rightarrow x^2 + x^2 + 4x + 4 = x^2 + 10x + 25 \rightarrow x^2 - 6x - 21 = 0$. Let's solve this equation by completing the square. Doing so, we have $x^2 - 6x + \underline{9} = 21 + \underline{9} \rightarrow (x - 3)^2 = 30 \rightarrow x - 3 = \pm\sqrt{30} \rightarrow x = 3 \pm \sqrt{30}$. Since x represents a side length of the triangle, it must be positive. Thus, $x =$ **3 + √30** or **√30 + 3** units.

Workout 8

241. First, we are told the value ratios of the coins are 1 sickle = 1/17 galleon and 1 knut = 1/29 sickle = 1/29 × 1/17 galleon. Then, we are given the quantity ratio of the number of galleons Ron has to the number of knuts and the quantity ratio of the number of sickles Ron has to the number of knuts. If we assume the number of knuts is 1, then the number of sickles must be 2, and the number of galleons must be 3. Converting these quantities to units of galleons, the 1 knut has a value of 1/29 × 1/17 galleon, the 2 sickles have a combined value of 2 × 1/17 galleon and then there are the 3 galleons. But we can't have any fractions of a galleon, since we are told that the total value N is an integer. Multiplying these quantities by 17 × 29 gives us values of (17 × 29) × 1/29 × 1/17 = 1 galleon's worth of knuts, (17 × 29) × 2 × 1/17 = 58 galleons' worth of sickles and (17 × 29) × 3 = 1479 galleons' worth of galleons. So, the least value of N is 1 + 58 + 1479 = **1538**. It should be noted that this minimum value is obtained by Ron having 29 × 17 × 1 = 493 knuts, 58 × 17 = 986 sickles and 1479 galleons. We confirm that there are twice as many sickles as knuts, and there are three times as many galleons as knuts.

242. Since the initial letter of these "words" must be A, E, I, O or U, there are 5 possibilities for the first letter. There are no restrictions on the next three letters, so there are 26 possibilities for each of those. The final letter cannot be one of the five vowels, so there are just 21 possibilities for the final letter. That gives us $5 \times 26 \times 26 \times 26 \times 21 =$ **1,845,480** words.

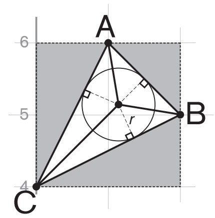
243. To calculate the number of positive integer divisors of 10!, rewrite it as a prime factorization: $10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = (2 \times 5) \times 3^2 \times 2^3 \times 7 \times (2 \times 3) \times 5 \times 2^2 \times 3 \times 2 = 2^8 \times 3^4 \times 5^2 \times 7^1$. Now we consider ways to "make" divisors of 10! by choosing combinations of these prime factors to multiply. With respect to the eight available instances of 2 as a factor, we actually have 9 different choices since we can choose to use 0, 1, 2, 3, 4, 5, 6, 7 or 8 of them. Similarly, there are 5 choices for the four instances of 3, 3 choices for the two instances of 5, and 2 choices for the single instance of 7. The number of positive integer divisors, then, is $9 \times 5 \times 3 \times 2 =$ **270** divisors.



244. If we move the chord of length 6, as shown, so that it shares an endpoint with the chord of length 8, we do not change the area of the shaded region, and we form a 6-8-10 right triangle with a hypotenuse that is a diameter of the circle. We now see that the shaded region consists of this right triangle and half of the circle, which has radius 5. The area of the shaded region, then, is $1/2 \times 6 \times 8 + 1/2 \times \pi \times 5^2 \approx$ **63.3** units^2.

245. For a best-of-five series to consist of exactly four games, the winning team must have exactly one loss, and the loss can't be the fourth game. The possibilities are LWWW, WLWW, or WWLW, for either team. The probability of the Anteaters winning any particular game is 60% = 0.6, which means that the probability of the Bears winning any particular game must be 100 − 60 = 40% = 0.4. The probability of the series consisting of exactly 4 games is the sum of the probabilities of each team winning the series in 4 games, which is $[(0.4 \times 0.6 \times 0.6 \times 0.6) + (0.6 \times 0.4 \times 0.6 \times 0.6) + (0.6 \times 0.6 \times 0.4 \times 0.6)] + [(0.6 \times 0.4 \times 0.4 \times 0.4) + (0.4 \times 0.6 \times 0.4 \times 0.4) + (0.4 \times 0.4 \times 0.6 \times 0.4)] = 3 \times (0.4 \times 0.6^3 + 0.6 \times 0.4^3) = 0.3744 \approx$ **37**%.

246. For this particular soccer field to go from the smallest-size regulation field to the largest-size regulation field would mean an increase in area of $[(110 \times 73) - (100 \times 64)] \div (100 \times 64) \approx$ **25**%.



247. The center of the inscribed circle is the incenter of the circumscribing triangle, which is the intersection of its vertex angle bisectors. As the figure shows, these angle bisectors divide $\triangle ABC$ into three triangles, each with a base that is a side of $\triangle ABC$. The heights of these three triangles are congruent, as they are all equal to the radius of the inscribed circle. It follows, then, that the area of $\triangle ABC$ is given by the formula $A = 1/2 \times (AB + BC + AC) \times r$. Notice that $\triangle ABC$ can be circumscribed by a 2-by-2 square. The area of $\triangle ABC$ is the area of this square minus the combined areas of the three shaded right triangles that surround $\triangle ABC$. So, $\triangle ABC$ has area $4 - (1 + 0.5 + 1) = 1.5$ units^2. Using the Pythagorean theorem, we can find the lengths of the sides of $\triangle ABC$. We have $AB = BC = \sqrt{(1^2 + 2^2)} = \sqrt{5}$, and $AC = \sqrt{(1^2 + 1^2)} = \sqrt{2}$. Substituting the side lengths and area for $\triangle ABC$ into the area formula above, we get $1.5 = 1/2 \times (\sqrt{5} + \sqrt{2} + \sqrt{5}) \times r \rightarrow r = 3/(2\sqrt{5} + \sqrt{2})$. Therefore, the inscribed circle has area $\pi \times [3/(2\sqrt{5} + \sqrt{2})]^2 \approx$ **0.82** units^2.

248. One way to solve this problem is by converting both numbers to base ten and multiplying. In base eight the place values are, from right to left, $8^0 = 1$, $8^1 = 8$, $8^2 = 64$, $8^3 = 512$, and so on. The base-ten value of 351_8 is $3 \times 64 + 5 \times 8 + 1 \times 1 = 233$. The base-ten value of 10_8 is $1 \times 8 + 0 \times 1 = 8$. So, we have $351_8 \times 10_8 = 233 \times 8 =$ **1864**. Alternatively, you might recognize that multiplying a base-eight number by 10_8 has the same effect as multiplying a base-ten number by 10. In other words, $351_8 \times 10_8 = 3510_8$. The base-ten value of 3510_8 is $3 \times 512 + 5 \times 64 + 1 \times 8 + 0 \times 1 =$ **1864**.

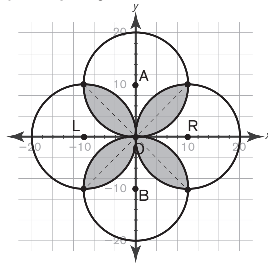
249. Evaluating the function for the values 2, 5 and 7, we get three equations, labeled [1], [2] and [3]:

$$\begin{aligned} 4a + 2b + c &= 3 \quad [1] \\ 25a + 5b + c &= -6 \quad [2] \\ 49a + 7b + c &= 10 \quad [3] \end{aligned}$$

There are numerous ways to solve this system of three equations and three unknowns. We will start by subtracting equation [1] from equation [2] to get $21a + 3b = -9 \rightarrow 7a + b = -3$. Next, we'll subtract equation [2] from equation [3] to get $24a + 2b = 16 \rightarrow 12a + b = 8$. We will label these new equations [4] and [5] as follows:

$$\begin{aligned} 7a + b &= -3 \quad [4] \\ 12a + b &= 8 \quad [5] \end{aligned}$$

With equations [4] and [5] we now have two equations in two variables, a and b. Subtracting equation [4] from equation [5], we get $5a = 11$, so $a = 11/5$. But we are asked to find the value of c. Let's try to get an equation in two variables a and c. We can add equations [1] and [2] to get $29a + 7b + 2c = -3$. Subtracting this result from equation [3] yields $20a - c = 13 \rightarrow c = 20a - 13$. Substituting for a gives us $c = 20 \times 11/5 - 13 =$ **31**.



250. There are four regions that are painted with two colors. In the diagram, each of these regions has been divided in half with a dotted line so that we have 8 congruent circle segments. Each of these segments is bounded by an arc that is 1/4 of the circumference of a circle of radius 10 units, and has area equal to $(1/4 \times \pi \times 10^2) - (1/2 \times 10 \times 10) = 25\pi - 50$. Since there are 8 of these segments that make up the regions painted with at least two colors, the total area is $8 \times (25\pi - 50) \approx$ **228** cm^2.

ANSWERS & PROBLEM INDEX

WARM-UP 12		WARM-UP 13		WARM-UP 14		WORKOUT 7		WORKOUT 8	
201. 2	(4)	211. 1/300,000	(3)	221. 0	(5)	231. 42/625	(5)	241. 1538	(4)
202. 60	(2)	212. 55,440	(5)	222. 121	(7)	232. 2.3	(6)	242. 1,845,480	(3)
203. 36	(4)	213. 4/3	(3)	223. $9\frac{5}{8}$	(3)	233. 11/4	(6)	243. 270	(5)
204. 42	(5)	214. 34	(3)	224. 29	(3)	234. 416	(6)	244. 63.3	(5)
205. 86	(5)	215. −10/101	(4)	225. 1/2	(5)	235. 1608	(4)	245. 37	(5)
206. 50	(5)	216. 6	(5)	226. 47	(5)	236. 768	(6)	246. 25	(3)
207. 1540	(5)	217. $6\sqrt{3}$	(6)	227. 30.25	(4)	237. 63	(6)	247. 0.82	(7)
208. 70	(5)	218. $5\sqrt{15}/9$	(5)	228. 243/1024	(5)	238. 127	(5)	248. 1864	(5)
209. −2	(6)	219. 11	(4)	229. $100\sqrt{2}$	(5)	239. 76	(4)	249. 31	(7)
210. 3	(4)	220. 9	(5)	230. 5	(5)	240. $3 + \sqrt{30}$ or $\sqrt{30} + 3$	(5)	250. 228	(5)

Al

ALGEBRAIC EXPRESSIONS & EQUATIONS

201	(4)	8.EE.1
206	(5)	8.EE.8
209	(6)	F-BF.3
218	(5)	N-RN.2
219	(4)	A-REI.7
221	(5)	F-FI.5
225	(5)	8.EE.8
230	(5)	F-IF.8
232	(6)	7.EE.2
240	(5)	8.G.7
249	(7)	F-IF.2

Pc

PROBABILITY, COUNTING & COMBINATORICS

207	(5)	S-CP.9
212	(5)	S-CP.9
224	(3)	SMP
228	(5)	S-CP.5
231	(5)	S-CP.2
236	(6)	S-CP.9
242	(3)	S-CP.9
245	(5)	S-CP.3

Nt

NUMBER THEORY

203	(4)	SMP
205	(5)	8.EE.1
210	(4)	6.NS.4
211	(3)	7.NS.2
216	(5)	6.NS.4
238	(5)	SMP
243	(5)	6.NS.4
248	(5)	SMP

Lo

LOGIC

214	(3)	SMP
222	(7)	SMP
237	(6)	SMP

Sd

STATISTICS

235	(4)	7.SP.2

Sp

SEQUENCES, SERIES & PATTERNS

215	(4)	7.EE.1
227	(4)	6.SP.5
229	(5)	8.G.8
234	(6)	F-IF.3

Pr

PROPORTIONAL REASONING

208	(5)	6.G.1
213	(3)	7.G.6
241	(4)	6.RP.3

Me

MEASUREMENT

223	(3)	7.G.6
246	(3)	7.G.6

Pf

PERCENTS & FRACTIONS

202	(2)	SMP
239	(4)	7.RP.3

Pg

PLANE GEOMETRY

204	(5)	7.G.6
217	(6)	G-SRT.5
244	(5)	G-C.2
250	(5)	G-C.5

Cg

COORDINATE GEOMETRY

220	(5)	G-GPE.7
226	(5)	G-GPE.7
233	(6)	G-SRT.5
247	(7)	8.G.8

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